

# Simultaneous arm morphing quadcopter and autonomous flight system design

*Oguz Kose*

Department of Aircraft Technology, Erzincan Binali Yildirim University, Erzincan, Turkey

*Tugrul Oktay*

Faculty of Aeronautics and Astronautics, Erciyes University, Kayseri, Turkey, and

*Enes Özen*

Department of UAV Technology, Hasan Kalyoncu University, Gaziantep, Turkey

## Abstract

**Purpose** – The purpose of this paper is to obtain values that stabilize the lateral and longitudinal flight of the quadrotor for which the morphing amount and the best Proportional-Integral-Derivative (PID) coefficients are determined by using the simultaneous perturbation stochastic approximation (SPSA) optimization algorithm.

**Design/methodology/approach** – Quadrotor consists of body and arms; there are propellers at the ends of the arms to take off and rotors that rotate them. By reducing the angle between mechanism 1 and the rotors with the horizontal plane, the angle between mechanism 2 and the arms, the rotors rise and different configurations are obtained. Conventional multi-rotor aircraft has a fixed fuselage and does not need a tail rotor to change course as helicopters do. The translational and rotational movements are provided by the rotation of the rotors of the aircraft at different speeds by creating moments about the geometric center in 6-degree-of-freedom (DOF) space. These commands sent from the ground are provided by the flight control board in the aircraft. The longitudinal and lateral flight stability and properties of different configurations evaluated by dynamic analysis and simulations in 6 DOF spaces are investigated. An algorithm and PID controller are being developed using SPSA to achieve in-flight position and attitude control of an active deformable aircraft. The results are compared with the results of the literature review and the results of the previous article.

**Findings** – With SPSA, the best PID coefficients were obtained in case of morphing.

**Research limitations/implications** – The effects of quadrotor arm height and hub angle changes affect flight stability. With the SPSA optimization method presented in this study, the attitude is quickly stabilized.

**Practical implications** – With the optimization method, the most suitable PID coefficients and angle values for the lateral and longitudinal flight stability of the quadrotor are obtained.

**Social implications** – The transition rate and PID coefficients are determined by using the optimization method, which is advantageous in terms of cost and practicality.

**Originality/value** – With the proposed method, the aircraft can change shape to adapt to different environments, and the parameters required for more stable flight for each situation will be calculated, and this will be obtained more quickly and safely with the SPSA optimization method.

**Keywords** Quadrotor, Morphing, SPSA, Optimization, PID

**Paper type** Research paper

## Nomenclature

### Definitions, acronyms and abbreviations

*PID* = Proportional integrator and derivative;

*SPSA* = Simultaneous perturbation stochastic approximation;

*ESC* = Electronic speed control;

*UAV* = Unmanned aerial vehicle;

*VTOL* = Vertical take-off and landing;

*DOF* = Degree of freedom; and

*TOW* = Take of weight

### Symbols

*x* = Position along *x*-axis;

*y* = Position along *y*-axis;

*z* = Position along *z*-axis;

*u* = Velocity in the *x*-axis direction;

*v* = Velocity in the *y*-axis direction;

*w* = Velocity in the *z*-axis direction;

*p* = Roll rate;

*q* = Pitch rate;

*r* = Yaw rate;

The current issue and full text archive of this journal is available on Emerald Insight at: <https://www.emerald.com/insight/1748-8842.htm>

*Research funding:* This work has been supported by Erciyes University Scientific Research Projects Coordination Unit under grant number FBA-2022-12375.



Aircraft Engineering and Aerospace Technology  
© Emerald Publishing Limited [ISSN 1748-8842]  
[DOI 10.1108/AEAT-05-2023-0146]

Received 25 May 2023  
Revised 22 July 2023  
Accepted 21 August 2023

$\phi$	= Roll angle;
$\theta$	= Pitch angle;
$\varphi$	= Yaw angle;
$g$	= Gravitational acceleration;
$I_x$	= Moment of inertia about $x$ -axis;
$I_y$	= Moment of inertia about $y$ -axis;
$I_z$	= Moment of inertia about $z$ -axis;
$\omega$	= Total propellers' speed;
$\omega_1$	= Front right propeller speed;
$\omega_2$	= Rear right propeller speed;
$\omega_3$	= Rear left propeller speed;
$\omega_4$	= Front left propeller speed;
$f_t = U_1$	= Vertical thrust factor;
$\tau_x = U_2$	= Rolling thrust factor;
$\tau_y = U_3$	= Pitching thrust factor;
$\tau_z = U_4$	= Yawing thrust factor;
$m$	= Quadrotor mass;
$b$	= Thrust factor;
$L$	= $l$ Length of arm;
$L1$	= The length of the 1st part of the arm;
$L2$	= The length of the 2st part of the arm
$d$	= Drag factor;
$\alpha$	= Hub angle;
$\beta$	= Arm rise angle;
$x(t)$	= State vector;
$y(t)$	= Output vector;
$u(t)$	= Input or control vector;
$A$	= System matrix;
$B$	= Input matrix;
$C$	= Output matrix;
$D$	= Feedforward matrix;
$R$	= Radius of quadrotor body circle;
$M_{arm}$	= Mass of quadrotor arm;
$\mathcal{J}$	= Cost index;
$\mathcal{J}_{long}$	= Cost index of longitudinal flight;
$\mathcal{J}_{lat}$	= Cost index of lateral flight;
$\mathcal{J}_{tot}$	= Total cost index;
$\mathcal{J}_{tot_i}$	= Total cost index of $i$ . iteration;
$T_{rt} = RT$	= Rise time;
$T_{rt_{long}} = RT_{long}$	= Rise time of longitudinal flight;
$T_{rt_{lat}} = RT_{lat}$	= Rise time of lateral flight;
$T_{st} = ST$	= settling time;
$T_{st_{long}} = ST_{long}$	= Settling time of longitudinal flight;
$T_{st_{lat}} = ST_{lat}$	= Settling time of lateral flight;
$OS$	= Overshoot;
$OS_{long}$	= Overshoot of longitudinal flight;
$OS_{lat}$	= Overshoot of lateral flight;
$D$	= Propeller diameter;
$n$	= Number of propellers;
$\rho$	= Air density

## Introduction

Unmanned aerial vehicles (UAVs) are classified as rotary-wing and fixed-wing aircraft. Electronic hardware and software technology is widely used in aircrafts. All information is instantly controlled from the ground control station thanks to the remote communication network (Bao *et al.*, 2022). Today, they are preferred because they are used for different tasks without risking the human factor – search and rescue

(Holzmann *et al.*, 2021). Robot operating system-based UAVs perform their assigned task autonomously (Gao *et al.*, 2018). Aircraft configurations are available for each mission profile. Fixed-wing aircraft pull the air backwards at high speed by turning the propeller in the longitudinal axis direction (Coban *et al.*, 2020). The resulting high-speed airflow passes under the wings and creates a pressure difference. Thus, the lift force is produced, which enables the aircraft to rise. Rotary-wing aircraft rotates the propellers along the vertical axis, carries the air to high speeds thanks to the rotors and creates upward thrust. The thrust begins to rise when it overcomes the weight of the aircraft. Advantages compared to fixed-wing aircraft; they can hover in the air (Falanga *et al.*, 2019). Disadvantages are limited to the controller to be controlled and task time is low as energy loss is high. Rotary-wing aircraft are named according to different configurations, rotors and their positions. In this study, the quadrotor was preferred due to its high manoeuvrability and simple mechanical structure. The isometric view of the quadrotor considered in this study is shown in supplementary material Figure 1.

In recent years, many studies have been carried out on UAVs in the scientific and military field. Quadrotor type UAVs, which are in the rotary wing category, stand out in these areas. What makes the quadrotor type UAV stand out from other UAVs is its structural simplicity. In addition, it can perform vertical take-off and landing (VTOL) without the need for a runway when taking off and landing, and it has high maneuverability with its four rotors. Because of these advantages, the quadrotor has become a popular and studied subject. These studies generally focused on quadrotor control. Mathewson and Fahimi (2023) discussed the nonlinear adaptive sliding mode control (NASMC) method for 2023 quadcopter applications. This control design aims to control the system of unknown parameters. Traditional model-based control methods require a dynamic mathematical model of the system with known parameters. Using NASMC, the definition of time-consuming system parameters can be skipped, and the next step can be quickly passed. In addition, once NASMC is defined for a quadrotor, it will be available for all other quadrotors. In this way, system delays will be avoided and the quadrotor will be controlled quickly and safely. Lee *et al.* (2012) used the PID control method for angular motion control and the dynamic surface control method for altitude control that a quadrotor system consists of two subsystems. The performance of the controller was examined in the simulation environment and compared on the basis of the Lyapunov stability theory. Yongqiang *et al.* (2012) developed a linearization-based controller for attitude, in which a quadrotor system consists of two subsystems. The effectiveness of the Tsinghua Autonomous Quadrotor System developed for this study using the classical PD method is presented with the results. Rozi *et al.* (2017) did control design and dynamic system modelling for quadrotor. When the dynamic system can be modeled close to the real system properties, accurate analysis and system modification can be easily obtained, and in this, they proposed the quaternion method to model the quadrotor behavior. Jithu and Jayasree (2016) present a research on modeling a quadrotor model using the Euler-Lagrange method. They stated that with this model, the kinetic and potential energy of the system will be obtained by modeling the external forces

obtained from the aerodynamic analysis, and a control strategy will be developed for the uncoupled states of the quadrotor by using the optimal control method.

The aim of the research is to develop an air robot that can change shape and shrink in closed environments, thus avoiding obstacles. It is to develop a more stable and controllable quadcopter that will both resist atmospheric noise and perform the task successfully by changing the hub angle and arm rise angle according to the type of mission in indoor and outdoor environments. For this purpose, determining the best PID coefficients (Can and Ercan, 2022), hub angle and arm rise angle is important for a stable and controllable flight. Simultaneous perturbation stochastic approximation (SPSA) optimization algorithm is used for quadrotor system to determine hub angle change and arm rise angle change with PID coefficients in lateral and longitudinal flight. The design of the simultaneous quadrotor lateral and longitudinally contractible quadrotor autopilot system and the design of the mass transit system are discussed together for the first time. Therefore, the angle between the arms and the arm rise angle parameters are determined by both lateral and longitudinal movement (Desbriez *et al.*, 2017). SPSA design coefficients are known to provide fast convergence and safe results compared to other algorithms (Kose and Oktay, 2022).

The contributions of this article can be summarized as follows:

- Quadrotor type UAV's both longitudinal and lateral flight control under the morphing effect was carried out in a simulation environment using SPSA and PID algorithms.
- A total of eight parameters were estimated using SPSA. These parameters are: two morphing ratios ( $\beta$  and  $\alpha$ ), three longitudinal flight PID gain, three lateral flight PID gain.
- Moments of inertia were calculated geometrically, depending on the morphing rates ( $\beta$  and  $\alpha$ ) estimated by SPSA.
- Estimating or determining variable parameters in UAV systems is a challenging task. Both the moments of inertia and the controller gain coefficients vary with the morphing discussed in this article. Coping with these variability and realizing a unique autopilot is the main objective of the study. From this point of view, this article has brought an innovative perspective in this field by combining the morphing effect under both the hub angle and the arm rise angle, contributing to the development of an independent autopilot in quadrotor control.

## Dynamic modelling

The quadrotor is a nonlinear six degree of freedom (DOF) system. The movements of the four propellers can be examined under two separate subheadings. These are translational and rotational dynamics (Shakeel *et al.*, 2022).

$$F = ma \quad (1)$$

$$M = Ia \quad (2)$$

Here,  $F$  is the total force applied to the centroid of the four propellers,  $m$  is the mass of the four propellers and can be

rewritten in the body frame as follows (Eraslan *et al.*, 2020a, 2020b).

$$F_i = b\omega_i^2; i = 1..4 \quad (3)$$

The four propellers on the quadrotor's body are rotated by the rotors, creating an anti-gravity force along the  $z$ -axis (Eraslan *et al.*, 2020a, 2020b). The equation given in equation (3), which is derived by accepting values such as the area swept by the propeller during rotation, and the density of the air, shows the force produced by each propeller. Angular velocity squared times  $b$  coefficient,  $b$  coefficient; is the coefficient obtained experimentally. All propellers and driving rotors of a quadrotor aircraft are equivalent. The up command transmitted to the controller ensures that all propellers rotate at the same speed. Derived from equations (1) and (2) and (3), and equation (4) expresses the total force produced. It enables the aircraft to perform linear motion. In case of  $U_1 > TOW$  (Take of Weight), the aircraft takes off. As the propellers are equivalent, there is no force in other directions; it is zero (Kose and Oktay, 2019a, 2019b).

$$U_1 = b(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \quad (4)$$

The quadrotor structure consists of  $x$ -,  $y$ - and  $z$ -axis on the fuselage axis located at the center of gravity of the  $x$  configuration aircraft. The axis along the  $x$ -axis is called the longitudinal axis, the axis along the  $y$ -axis is called the lateral, and the axis along the  $z$ -axis is called the vertical axis. Equation (7) indicates the force produced by the aircraft in the  $z$ -axis. Suspended in air when the force is equal to the total weight. The forward movement transmitted to the controller causes the rotor speeds of the aircraft to change and allows it to move forward. Equation (6) indicates forward motion. Rotational motion around the lateral axis and forward linear motion are performed together. For lateral movement, lateral movement occurs when the speeds of the rotors on the right with respect to the  $x$ -axis are different from the speeds of the rotors on the left. Equation (5) indicates lateral movement.

$$U_2 = b \frac{L}{\sqrt{2}} (-\omega_1^2 + \omega_2^2 + \omega_3^2 - \omega_4^2) \quad (5)$$

$$U_3 = b \frac{L}{\sqrt{2}} (\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2) \quad (6)$$

SahinbeyQuad has the ability to change both arm raising and hub angle on  $x$ - and  $y$ -axes (Oktay and Kose, 2019). Arm raising and hub angle change are named as configurations 1, 2 and 3 and are shown in supplementary material Figures 2, 3 and 4.

The changes in the geometric structure of the resulting deformation are given in Table 1.

As seen in equation (5), the distance of the center of the rotor to the center of gravity is effective in forward and lateral movements. These values will be taken into account when designing the controller in the next section.

The aircraft's upward movement and yaw movement are independent of the aircraft's shape change. The rotation movement in the  $z$ -axis is called yaw. It is used to determine the

Table 1 Quadrotor geometry

Quadrotor configuration	Width(mm)/Length(mm)
Configuration 1	375/375
Configuration 2	265/459
Configuration 3	246/427

Source: Table by authors

route of the aircraft. For this, the drag force specified in equation (7) is used (Wallace, 2016).

$$Q_i = d\omega_i^2; \quad i = 4 \quad (7)$$

In equation (7),  $d$  is the friction force coefficient. The rotation of the rotors causes the drag force to be generated and the torque opposite to the direction of rotation of the rotor. The intensity of the torque is the product of its angular velocity squared and  $d$  (drag coefficient). The two rotors of the quadrotor rotate clockwise (CW), while the other two rotate counter-clockwise (CCW) (Oktay and Sal, 2016). So the tool can be controlled around the  $z$ -axis. This situation is given in equation (8).

$$U_4 = d(\omega_1^2 - \omega_2^2 - \omega_3^2 - \omega_4^2) \quad (8)$$

Quadrotor is a six-degree-of-freedom aircraft. Entry members were given at equations (4), (5), (6) and (8) output stage. This road tool has missing handler (Bai and Gururajan, 2019). The  $x$  and  $y$  commands needed are generated by the controller.

### Autopilot system and control algorithm

Designing a control system is a fundamental process when implementing any system or model. The control system produces the behavior and responses of the model based on various mathematical laws. The mathematical model of an UAV includes control surfaces, responses under different conditions and behaviors that occur in its geometry (Christiansen, 2013). The mathematical model of a UAV is required because real-time testing is costly, and airworthy environments are difficult to create. Thus, the model becomes controllable, and its purpose is achieved. The model of the UAV consists of mathematical expressions, and these include kinematic and dynamic equations. State space model representation is used to make the model useful. The state space model, on the other hand, is expressed in matrix form as a combination of first-order differential equations and input, output and state variables (Tahir *et al.*, 2019). Quadrotor model consists of nonlinear equations. In this study, a linear model was used to work in harmony with the control algorithm and optimization method. In addition, it is easy to convert the linear model to the state space model. The linear model can be derived from the nonlinear model by considering the small perturbation theory and a given flight situation conditions. In this study, a linear model was obtained by considering the quadrotor hover flight situation. In the linear model, the longitudinal and lateral flight equations are separated from each other, and a separate but integrated model is created for

the two flight situations. Accordingly, the longitudinal flight linear model can be shown as follows.

$$\dot{x} = u \quad (9a)$$

$$\dot{z} = w \quad (9b)$$

$$\dot{u} = -g\theta \quad (9c)$$

$$\dot{w} = \frac{U_1}{m} \quad (9d)$$

$$\dot{q} = \frac{U_3}{I_x} \quad (9e)$$

$$\dot{\theta} = q \quad (9f)$$

The lateral flight linear model can be shown as follows:

$$\dot{y} = v \quad (10a)$$

$$\dot{v} = g\phi \quad (10b)$$

$$\dot{p} = \frac{U_2}{I_y} \quad (10c)$$

$$\dot{r} = \frac{U_4}{I_z} \quad (10d)$$

$$\dot{\phi} = p \quad (10e)$$

$$\dot{\psi} = r \quad (10f)$$

The obtained mathematical equations show that it is a four-propeller, multi-input, multi-output nonlinear system, as in many contemporary systems. This makes the system more complex to understand. To examine such a system and to reduce the processing load of the designer, state space representation is used to express the internal dynamics of the system, namely, the state variables, with first-order derivative equations (Sahin and Oktay, 2017).

$$\dot{x} = Ax(t) + Bu(t),$$

$$y = Cx(t) + Du(t). \quad (11)$$

In equation (11),  $x$  denotes the state vector and  $u$  the control vector. The state vector and control vector of the four propellers are as given below.

$$x = [x \quad \dot{x} \quad y \quad \dot{y} \quad z \quad \dot{z} \quad \phi \quad \dot{\phi} \quad \theta \quad \dot{\theta} \quad \varphi \quad \dot{\varphi}]^T \quad (12a)$$

$$u = [U_1 \quad U_2 \quad U_3 \quad U_4]^T \quad (12b)$$

The longitudinal flight state space representation obtained for the control of four propellers using equations (12a) and (12b)

can be obtained as given in equations (13) and (14). In the longitudinal state space model output matrix, the outputs followed in this study are given as outputs, as  $z$  (upward) and  $\theta$  (pitch) are followed [equations (13) and (14)] (Kose and Oktay, 2019a, 2019b).

$$\begin{bmatrix} \dot{x} \\ \dot{z} \\ \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -g \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ z \\ u \\ w \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1/m & 0 \\ 0 & 1/I_y \\ 0 & 0 \end{bmatrix} \begin{bmatrix} U_1 \\ U_3 \end{bmatrix} \quad (13)$$

$$y = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ z \\ u \\ w \\ q \\ \theta \end{bmatrix} \quad (14)$$

The lateral flight state space model is given as output because  $\varnothing$  (roll) and  $\varphi$  (yaw) are followed in the output matrix [equations (15) and (16)] (Çoban and Oktay, 2018).

$$\begin{bmatrix} \dot{y} \\ \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & g & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ v \\ p \\ r \\ \phi \\ \varphi \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1/I_x & 0 \\ 0 & 1/I_z \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} U_2 \\ U_4 \end{bmatrix} \quad (15)$$

$$y = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ v \\ p \\ r \\ \phi \\ \varphi \end{bmatrix} \quad (16)$$

In classical control, phase forward or reverse compensators are designed, and a pole and zero are added to the system (Oktay et al., 2017). The reason for this is that it has proven itself in its uncomplicated structure, ease of application and control. PID is a specialized version of these. It is effective on both transient and steady state response of the system. It is a controller that is frequently used in the control field today. The structure and control rule of the PID controller are given in supplementary material Figure 5 and equation (17) (Çoban et al., 2019).

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de(t)}{dt}, \quad (17)$$

$$e(t) = y_r(t) - y(t).$$

PID (proportional integral derivative) generates a new signal by processing the difference between the input and output signals, that is, the error signal (multiplying with the proportional gain, taking the integral and derivative). Thus, the system can behave

close to the desired behavior. If we examine the PID controller briefly, the waveform of the system does not change in the proportional part, only the error signal is strengthened and weakened. In the integral, the steady state error of the system is corrected, it does this thanks to its movement in the form of the sum of the errors, slows the response of the system and drags it toward the unstable region. The derivative is related to change in nature, it shortens the response of the transient state by intervening at the points where the change exists in the error, it has no effect on that issue as there is no change in the steady state error (Zhou et al., 2019).

## Morphing and moment of inertia

Morphing can generally be described as changes in the geometry of the quadrotor or UAV before or during flight (Kose and Oktay, 2023). Depending on the type of UAV, methods such as lengthening and shortening the arms or changing the hub angle can be used for morphing. In this study, morphing was performed by considering both hub angle and arm rise angle together. In the morphing state, while the mass of the quadrotor remains constant, the moment of inertia changes as the distance of the rotors from the rotation axis changes (Oktay and Kose, 2020). In addition, as morphing is performed during flight, the moment of inertia and simulations have been controlled considering this situation. Accordingly, the change in moments of inertia at the moment of morphing during flight can be explained in the following order.

As shown in supplementary material Figure 6, the quadrotor is assumed to consist of four arms arranged perpendicular to each other.

The moment of inertia of an arm (rod) of length  $L$  is calculated by equation (18).

$$I = \frac{1}{3} mL^2 \quad (18)$$

As shown in supplementary material Figure 7, the quadrotor is assumed to be four equal-mass arms placed symmetrically at the center of mass.

In supplementary material Figure 7, the mass of the bar with  $m$  is given in kg and the length  $L$  in mm. Therefore, the moment of inertia about the  $z$ -axis is calculated as in equation (19).

$$I_z = \frac{4}{3} mL^2; L = L_1 + L_2 \cos \beta \quad (19)$$

As stated in equation (19), each arm is articulated and consists of two parts. As it moves  $L_2$  upwards by the actuator, the length of the arm becomes shorter as the arm rise angle ( $\beta_{\text{initial}} = 15^\circ$ ) increases and lengthens as it decreases. Calculation of moments of inertia about  $y$ -axis is given in equation (20).

$$I_y = \frac{1}{3} mb^2; b = L \cos \left( \frac{\alpha}{2} \right) \quad (20)$$

ŞahinbeyQuad activates the anterior and posterior angles between the arms equally by means of the actuator. As the angle between the arms ( $\alpha_{\text{initial}} = 75^\circ$ ) increases and decreases, the moment of inertia with respect to the  $y$ -axis changes. The same is true for the moment of inertia about the  $x$ -axis.

$$I_x = \frac{1}{3}ma^2; \quad a = L\sin\left(\frac{\alpha}{2}\right) \quad (21)$$

The shape change of the aircraft was determined to create three different body structures. In trunk structure 1, the hub angle ( $\alpha$ ) and the arm rise angle ( $\beta$ ) were accepted as ( $90^\circ$ ,  $0^\circ$ ), respectively. Belly angle ( $\alpha$ ) and arm rise angle ( $\beta$ ) were accepted as ( $60^\circ$ ,  $0^\circ$ ), respectively, in trunk structure (Configuration) 2. In the trunk structure (Configuration) 3, the hub angle ( $\alpha$ ) and the arm rise angle ( $\beta$ ) were accepted as ( $60^\circ$ ,  $30^\circ$ ), respectively. Changes in geometric measurements are given in Table 1.

In Configuration 3, both the hub angle and the arm rise angle change. In this case, the arm rise angle parameter is added to equations (20) and (21).

$$\begin{aligned} I_x &= \frac{1}{3}m\left((L1 + (L2\cos\beta))\sin\left(\frac{\alpha}{2}\right)\right)^2, \\ I_y &= \frac{1}{3}m\left((L1 + (L2\cos\beta))\cos\left(\frac{\alpha}{2}\right)\right)^2, \\ I_z &= \frac{1}{3}m(L1 + (L2\cos\beta))^2. \end{aligned} \quad (22)$$

The moment of inertia values of ŞahinbeyQuad in three different configurations are shown in Table 2.

### Simultaneous perturbation stochastic approximation optimization

To calculate the maximum flight performance of the quadrotor, it is necessary to establish and calculate the relationship between the PID coefficients, alpha and beta values. SPSA optimization method was used to calculate the values. SPSA is a method used for optimization in systems with more than one variable. SPSA is an algorithmic method for system optimization with many unknown parameters (Spall, 1998). SPSA has been successfully applied for predictive control and model-independent signal timing, simulation optimization and atmospheric modeling, where SPSA gives faster results compared to other optimization methods (Song et al., 2008). As a result, SPSA can provide high efficiency for large-scale problems.

Quadrotor arm rising, hub angle change and both longitudinal and lateral PID coefficients are independent parameters. SPSA working for the optimization of these parameters works as follows:

$$\hat{\theta}_{k+1} = \hat{\theta}_k - a_k \hat{g}(\hat{\theta}_k) \quad (23)$$

To estimate  $\hat{g}_k$ , two new parameters are obtained. Each  $\hat{\theta}_k$  is calculated mutually and independently. Average zero random variables  $\Delta_k$  value multiplied by a positive scalar  $c_k$ .

**Table 2** The moment of inertia values of ŞahinbeyQuad in three different configurations

ŞahinbeyQuad configuration	$I_x$ (kg/m <sup>2</sup> )	$I_y$ (kg/m <sup>2</sup> )	$I_z$ (kg/m <sup>2</sup> )
Configuration 1	0.0087781	0.0087781	0.0175562
Configuration 2	0.0043890	0.0131671	0.0175562
Configuration 3	0.0037281	0.0111844	0.0149125

Source: Table by authors

$$\hat{\theta}_k^+ = \hat{\theta}_k + c_k \Delta_k \quad (24)$$

$$\hat{\theta}_k^- = \hat{\theta}_k - c_k \Delta_k \quad (25)$$

For the cost function  $L$ , the estimate  $\hat{g}(\hat{\theta}_k)$ ;

$$\hat{g}(\hat{\theta}_k) = \begin{bmatrix} \frac{L(\hat{\theta}_k^+) - L(\hat{\theta}_k^-)}{2c_k \Delta_{k1}} \\ \vdots \\ \frac{L(\hat{\theta}_k^+) - L(\hat{\theta}_k^-)}{2c_k \Delta_{kp}} \end{bmatrix} \quad (26)$$

The angle between the quadrotor arms is adjusted according to the estimated shape change in the rise angle of the arms. The change of alpha and beta angles causes the moment of inertia to change. The values given in Table 1 give the values formed after the shape change. With optimization, the most suitable value for angle values is determined. This is necessary for control and stability (Kose and Oktay, 2021).

SPSA provides optimal results. SPSA offers faster results than other optimization methods (Sariff and Ismail, 2021). SPSA uses the cost function, which is a combination of performance criteria such as rise time, settling time and maximum overshoot system values for optimum results. A total of eight parameters are calculated with SPSA. These are three PID Longitudinal, three PID Lateral, alpha and beta angles. The aim is to minimize the cost index. The cost function is given in equation (23).

$$\mathcal{J} = \mathcal{J}_{lon} + A\mathcal{J}_{lat}; \quad A = \mathcal{J}_{Lon-initial} \quad (27)$$

The total cost index is considered together for both longitudinal flight and lateral flight. The cost function acts as a function of the cost index, varying according to the PID gains obtained by estimating the  $\alpha$  and  $\beta$  values. With the variation of PID gains, a performance review is made by considering the design performance criteria. The design performance criteria then give the total cost index in both longitudinal and lateral flight. When considered separately for longitudinal and lateral flight, the cost indices can be shown as follows:

$$\text{Cost}_{long} = RT_{long} + ST_{long} + OS_{long} \quad (28)$$

$$\text{Cost}_{lat} = RT_{lat} + ST_{lat} + OS_{lat} \quad (29)$$

The total cost index gives us information about the progress of the overall optimization. The total cost index can be shown as follows:

$$\% \mathcal{J}_{tot_i} = \mathcal{J}_{lon_i} + \frac{\mathcal{J}_{lon_0}}{\mathcal{J}_{lat_0}} + \mathcal{J}_{lat_i} \quad (30)$$

The zero state is found as follows. The initial values of the recommended autopilot were accepted as the initial value in this study. The PID value is (50, 5, 50). It is 75 for  $\alpha$  and 15 for  $\beta$ . The values that  $\alpha$  can take are  $60 < \alpha < 90$ ,  $\alpha_{start} = 75^\circ$ .

$$\frac{60}{75} \langle \alpha_{multiplier} \frac{90}{75} \rangle \quad (31)$$

The values that  $\beta$  can take are  $0 < \beta < 30$ :

$$\frac{0}{15} \langle \beta_{multiplier} \frac{30}{15} \rangle \quad (32)$$

Initial values of the parameters to be calculated with SPSA; they are  $K_{plon}$ ,  $K_{ilon}$ ,  $K_{Dlon}$  (25,2.5,25),  $K_{plat}$ ,  $K_{ilat}$ ,  $K_{Dlat}$  (25, 2.5, 25),  $\alpha$  (75°),  $\beta$  (15°).

## Results

Active deformation resulting from the elevation of the arms and the change in the umbilical angle were examined together for lateral and longitudinal flight. Calculations are for forward motion for 1 degree pitch and 1 degree roll angle for lateral motion. The cost index includes lateral and longitudinal flight terms. The combined approach was used instead of the traditional approach to minimize the cost index. Lateral and longitudinal flight control system was designed simultaneously to determine the optimal value of the conversion parameters. As a result, the longitudinal and lateral flight parameters are optimal. Supplementary material [Figures 8, 9, 10 and 11](#) show the total cost, variations of longitudinal PID parameters and variations of lateral PID parameters, respectively.

The shape change ([Oktay and Özen, 2022](#)) narrows laterally as the hub angle decreases from 90 degrees to 60 degrees in the initial state, and the variations of the angle values are given in supplementary material [Figure 12](#).

The shape change narrows laterally and longitudinally as the arm rise angle ( $\beta$ ) increases from 0 to 30 degrees in the initial state, and the variations of the angle values are given in supplementary material [Figure 13](#).

Indoor, closed-loop responses for both lateral and longitudinal flight were studied and are given in supplementary material [Figures 14 and 15](#), respectively. Supplementary material [Figure 14](#) shows the closed-loop simulation of the longitudinal motion of the quadcopter. In the simulation, arm rising, hub angle and PID coefficients obtained from SPSA were requested to follow a 1° trajectory. Supplementary material [Figure 15](#) shows the closed-loop simulation of the lateral motion of the quadrotor. In the simulation, arm rising, hub angle and PID coefficients obtained from SPSA were requested to follow a 1° trajectory. Trajectories given in both longitudinal and lateral motion have been successfully tracked. Outputs such as angular and linear speeds do not show catastrophic behavior.

The design performance criteria were examined separately to obtain improvements in the lateral and longitudinal flight autonomous performance of the quadrotor. Precise information about the stability of a system is provided by the design performance criteria. In supplementary material [Figures 16 and 17](#), the design performance criteria are presented for lateral and longitudinal flight.

## Conclusion

This paper discusses simultaneous lateral and longitudinal flight control system model for hub angle and arm rise angle.

The SPSA optimization method is used to calculate lateral and longitudinal flight gains with optimal transformation parameters. Using the optimization method SPSA, 41% of the total cost index is recorded on the assumed basis. The overall cost index performs well in lateral and longitudinal flight. In the lateral and longitudinal flight initial state, the PID coefficients are  $P = 50$ ,  $I = 5$  and  $D = 50$ .  $P = 40$ ,  $I = 5$  and  $D = 60$  for longitudinal flight from SPSA,  $P = 59$ ,  $I = 5$  and  $D = 59$  for lateral flight. Closed loop responses are studied in the closed environment for lateral and longitudinal flight. The desired trajectory of 1 degree for lateral flight and 1 degree for longitudinal flight was successfully followed.

In this study, after the dynamic model of the quadrotor was created, the moments of inertia obtained as a result of the change in the hub angle and the change in the arm rise angle are given in [Table 2](#). The obtained equations and results were simulated in the MATLAB/Simulink program using the state space model. By adding SPSA optimization method, alpha and beta angles are obtained for the most stable flight. The resulting alpha value is 89.8 degrees and beta is 12.3 degrees. In the simulation scopes, it was seen that the values obtained for grounded flight were reached in the first iteration.

As the performance criteria are related to how fast and stable the aircraft is to the reference point, lateral and longitudinal flight performance parameters (Rise Time, Settling Time, Overshoot) in the simulation results examined were found to be at an acceptable level by examining previous studies ([Şahin et al., 2022](#)).

As a result of the optimization analyses, the quadrotor showed similar performance values in lateral and longitudinal movements. The rise time from the performance parameters was 0.00031973 s for longitudinal flight in the first iteration. He showed a stable behavior at this value. From the performance parameters, the settling time was 0.00056885 s for longitudinal flight in the first iteration. Close values were obtained in lateral flight. The results show the SahinbeyQuad's alpha and beta values for the most stable flight during flight. Quadrotor status after SPSA optimization supplementary material is shown in [Figure 18](#).

## References

- Bai, Y. and Gururajan, S. (2019), "Evaluation of a baseline controller for autonomous 'figure-8' flights of a morphing geometry quadcopter: flight performance", *Drones*, Vol. 3 No. 3, pp. 1-36, doi: [10.3390/drones3030070](https://doi.org/10.3390/drones3030070).
- Bao, X., Niu, Y., Li, Y., Mao, J., Li, S., Ma, X., Yin, Q. and Chen, B. (2022), "Design and kinematic analysis of cable-driven target spray robot for citrus orchards", *Applied Sciences*, Vol. 12 No. 18, doi: [10.3390/app12189379](https://doi.org/10.3390/app12189379).
- Can, M.S. and Ercan, H. (2022), "Real-time tuning of PID controller based on optimization algorithms for a quadrotor", *Aircraft Engineering and Aerospace Technology*, Vol. 94 No. 3, pp. 418-430, doi: [10.1108/AEAT-06-2021-0173](https://doi.org/10.1108/AEAT-06-2021-0173).
- Christiansen, R.S. (2013), "Design of an autopilot for small unmanned aerial vehicles", *Journal of Chemical Information and Modeling*.
- Coban, S., Bilgic, H.H. and Akan, E. (2020), "Improving autonomous performance of a passive morphing fixed wing

- UAV”, *Information Technology And Control*, Vol. 49 No. 1, pp. 28-35, doi: [10.5755/j01.itc.49.1.23275](https://doi.org/10.5755/j01.itc.49.1.23275).
- Çoban, S. and Oktay, T. (2018), “Simultaneous design of a small UAV (unmanned aerial vehicle) flight control system and lateral state space model”, *Journal of Aviation*, Vol. 2 No. 2, pp. 70-76, doi: [10.30518/jav.461365](https://doi.org/10.30518/jav.461365).
- Çoban, S., Bilgiç, H.H. and Oktay, T. (2019), “Designing, dynamic modeling and simulation of ISTEPCOPTER”, *Journal of Aviation*, Vol. 3 No. 1, pp. 38-44.
- Desbiez, A., Expert, F., Boyron, M., Dipieri, J., Viollet, S. and Ruffier, F. (2017), “X-Morf: a crash-separable quadrotor that morfs its X-geometry in flight To cite this version : HAL id : hal-01644528”.
- Eraslan, Y., Ozen, E. and Oktay, T. (2020a), “A literature review on determination of quadrotor unmanned aerial vehicles propeller thrust and power coefficients”, *EğONS 10th International Conference on Mathematics, Engineering, Natural Medical*, GA, pp. 1-10.
- Eraslan, Y., Ozen, E. and Oktay, T. (2020b), “Effect of change in angle Between rotor arms on trajectory tracking quality of a PID controlled quadcopter”, *EğONS 10th International Conference on Mathematics, Engineering, Natural Medical*. GA, pp. 50-60.
- Falanga, D., Kleber, K., Mintchev, S., Floreano, D. and Scaramuzza, D. (2019), “The foldable drone: a morphing quadrotor that can squeeze and fly”, *IEEE Robotics and Automation Letters*, Vol. 4 No. 2, pp. 209-216, doi: [10.1109/LRA.2018.2885575](https://doi.org/10.1109/LRA.2018.2885575).
- Gao, Y., Du, Z., Gao, X., Su, Y., Mu, Y., Sun, L.N. and Dong, W. (2018), “Implementation of open-architecture kinematic controller for articulated robots under ROS”, *Industrial Robot: An International Journal*, Vol. 45 No. 2, pp. 244-254, doi: [10.1108/IR-09-2017-0166](https://doi.org/10.1108/IR-09-2017-0166).
- Holzmann, P., Wankmüller, C., Globocnik, D. and Schwarz, E.J. (2021), “Drones to the rescue? Exploring rescue workers’ behavioral intention to adopt drones in Mountain rescue missions”, *International Journal of Physical Distribution & Logistics Management*, Vol. 51 No. 4, pp. 381-402, doi: [10.1108/IJPDLM-01-2020-0025](https://doi.org/10.1108/IJPDLM-01-2020-0025).
- Jithu, G. and Jayasree, P.R. (2016), “Quadrotor modelling and control”, *International Conference on Electrical, Electronics, and Optimization Techniques, ICEEOT 2016*, pp. 1167-1172, [10.1109/ICEEOT.2016.7754868](https://doi.org/10.1109/ICEEOT.2016.7754868).
- Kose, O. and Oktay, T. (2019a), “Dynamic modeling and simulation of quadrotor for different flight conditions”, *European Journal of Science and Technology*, No. 15, pp. 132-142.
- Kose, O. and Oktay, T. (2019b), “Non simultaneous morphing system desing for quadrotors”, Vol. 16, pp. 577-588, doi: [10.31590/ejosat.569785](https://doi.org/10.31590/ejosat.569785).
- Kose, O. and Oktay, T. (2021), “Quadrotor flight system design using collective and differential morphing with SPSA and ANN”, *Nternational Journal of Intelligent Systems and Applications in Engineering (IjISAE)*, Vol. 9 No. 4, pp. 159-164.
- Kose, O. and Oktay, T. (2022), “Hexarotor yaw flight control with SPSA, PID algorithm and morphing”, *International Journal of Intelligent Systems And Applications in Engineering*, Vol. 10 No. 2, pp. 216-221, doi: [10.1039/b000000x](https://doi.org/10.1039/b000000x).
- Kose, O. and Oktay, T. (2023), “Simultaneous design of morphing hexarotor and autopilot system by using deep neural network and SPSA”, *Aircraft Engineering and Aerospace Technology*, Vol. 95 No. 6, pp. 939-949, doi: [10.1108/AEAT-07-2022-0178](https://doi.org/10.1108/AEAT-07-2022-0178).
- Lee, K.U., Kim, H.S., Park, J.B. and Choi, Y.H. (2012), “Hovering control of a quadrotor”, *International Conference on Control, Automation and Systems*, pp. 162-167.
- Mathewson, R. and Fahimi, F. (2023), “Nonlinear adaptive sliding mode control with application to quadcopters”, *Nonlinear Engineering*, Vol. 12 No. 1, doi: [10.1515/nleng-2022-0268](https://doi.org/10.1515/nleng-2022-0268).
- Oktay, T. and Kose, O. (2019), “Non simultaneous morphing system design for yaw motion in quadrotors”, *Journal of Aviation*, Vol. 3 No. 2, pp. 81-88.
- Oktay, T. and Kose, O. (2020), “Simultaneous quadrotor autopilot system and collective morphing system design”, *Aircraft Engineering and Aerospace Technology*, Vol. 92 No. 7, pp. 1093-1100, doi: [10.1108/AEAT-01-2020-0026](https://doi.org/10.1108/AEAT-01-2020-0026).
- Oktay, T. and Özen, E. (2022), “Effects of shape changing of morphing rotary wing aircraft on longitudinal and lateral flight”, *Journal of Aviation*, Vol. 6 No. 3, pp. 251-259, doi: [10.30518/jav.1080139](https://doi.org/10.30518/jav.1080139).
- Oktay, T. and Sal, F. (2016), “Combined passive and active helicopter main rotor morphing for helicopter energy save”, *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, Vol. 38 No. 6, pp. 1511-1525.
- Oktay, T., Uzun, M., Çelik, H. and Konar, M. (2017), “Pid based hierarchical autonomous system performance maximization of a hybrid unmanned aerial vehicle (huav)”, *Anadolu University Journal of Science and Technology A – Applied Sciences and Engineering*, Vol. 18 No. 2, pp. 1-1, doi: [10.18038/aubtda.322137](https://doi.org/10.18038/aubtda.322137).
- Rozi, H.A., Susanto, E. and Dwibawa, I.P. (2017), “Quadrotor model with proportional derivative controller”, *ICCREC 2017 - 2017 International Conference on Control, Electronics, Renewable Energy, and Communications, Proceedings, 2017-Janua*, pp. 241-246, [10.1109/ICCREC.2017.8226676](https://doi.org/10.1109/ICCREC.2017.8226676).
- Sahin, H. and Oktay, T. (2017), “Powerplant system design for unmanned tricopter”, *The Eurasia Proceedings of Science, Technology, Engineering & Mathematics*, Vol. 1, pp. 9-21.
- Sariff, N. and Ismail, Z.H. (2021), “Broadcast event-triggered control scheme for multi-agent rendezvous problem in a mixed communication environment”, *Applied Sciences*, Vol. 11 No. 9, doi: [10.3390/app11093785](https://doi.org/10.3390/app11093785).
- Shakeel, T., Arshad, J., Jaffery, M.H., Rehman, A.U., Eldin, E. T., Ghamry, N.A. and Shafiq, M. (2022), “A comparative study of control methods for X3D quadrotor feedback trajectory control”, *Applied Sciences*, Vol. 12 No. 18, doi: [10.3390/app12189254](https://doi.org/10.3390/app12189254).
- Song, Q., James, C.S. and Yeng Chai Soh, J.N. (2008), “Robust neural network tracking controller using simultaneous perturbation stochastic approximation”, in”, *Ieee Transactions ON NEURAL NETWORKS*, Vol. 19 No. 5, pp. 817-835.
- Spall, J.C. (1998), “An overview of the simultaneous perturbation method”, *Johns Hopkins Apl Technical Digest*, Vol. 19 No. 4, pp. 482-492, [www.jhuapl.edu/SPSA/PDF-SPSA/Spall\\_An\\_Overview.PDF](http://www.jhuapl.edu/SPSA/PDF-SPSA/Spall_An_Overview.PDF).

- Şahin, H., Kose, O. and Oktay, T. (2022), “Simultaneous autonomous system and powerplant design for morphing quadrotors”, *Aircraft Engineering and Aerospace Technology*, Vol. 94 No. 8, pp. 1228-1241, doi: [10.1108/AEAT-06-2021-0180](https://doi.org/10.1108/AEAT-06-2021-0180).
- Tahir, Z., Tahir, W. and Liaqat, S.A. (2019), “State space system modeling of a quad copter UAV”, arXiv preprint arXiv:1908.07401 [Preprint]
- Wallace, D.A. (2016), “Dynamics and control of a quadrotor with active geometric morphing”.
- Yongqiang, B., *et al.* (2012), “Robust control of quadrotor unmanned air vehicles”, *Chinese Control Conference*, Vol. CCC, pp. 4462-4467.

- Zhou, Y., Jinzhong, Z., Xiao, Y. and Ying, L. (2019), “Optimization of PID controller based on water wave optimization for an automatic voltage regulator system”, *Information Technology And Control*, Vol. 48 No. 1, pp. 160-171, doi: [10.5755/j01.itc.48.1.20296](https://doi.org/10.5755/j01.itc.48.1.20296).

### **Supplementary material**

The supplementary material for this article can be found online.

### **Corresponding author**

**Oguz Kose** can be contacted at: [oguzkose24@gmail.com](mailto:oguzkose24@gmail.com)