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ORIGINAL PAPER

Determination of univariate, bivariate and conditional return periods of hydrological droughts using two-dimensional multivariate functions

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Abstract

Hydrological drought is frequently considered as the most intricate type of drought because of its long-term impacts, interactions with human activities and slow response of water systems to climatic fluctuations. Drought severity and duration are key parameters to characterize the drought. This research models the conditional probabilities, univariate, bivariate and conditional return periods of drought severity and duration using two-dimensional multivariate copulas functions. The analyses use streamflow drought index (SDI) and mean monthly streamflow records of 24 stations in Yeşilırmak Basin, Turkey for a 3-month time scale. Drought characteristics are derived from Yevjevic's Run Theory. The dependence between drought characteristics is examined with non-parametric Kendall's τ. The best marginal distributions of drought characteristics are selected among six types of distributions based on goodness of fit tests. The best copulas which have been utilized for the modelling of conditional probabilities, univariate, bivariate and conditional return periods are defined across 10 types of copula functions based on tail dependence and goodness of fit tests. Results have indicated a strong correlation between drought severity and duration. Marginal distributions of drought characteristics are modelled by Lognormal and Weibull distribution mostly. In most of the stations Gumbel copula has been detected as the best copula. Results have put forth that, many parts of the basin have the risk of hydrological drought under different conditions. Therefore, the basin needs an effective drought management plan.

Keywords Hydrological drought · Streamflow drought index · Copula functions · Trend · Yeşilırmak · Turkey

1 Introduction

During history, water has become primary and an essential source for life of livings and the sustainability of environment and ecology (Mishra and Singh 2010; Rahmi et al. 2025; Li et al. 2025). As the key element and primary reason of hydrologic cycle, the water effects the sectors of drinking water, agriculture, energy, irrigation, industry, construction, transportation and logistics. It has been stated in the United Nations World Development Report (United Nations 2024) that worldwide freshwater withdrawals globally are predominantly for agriculture, representing 70%, followed by industrial uses at just under 20% and domestic use accounting for 12%, however these numbers vary widely depending on the level of economic development. While in high-income countries a larger part of water is used for industry, in lower-income countries the majority of their water (90% or higher) is used for agriculture. Out of the factors of usage areas of water and increasing population, the excessive usage of water that may lead to depletion of water, environmental damage, economic consequences is another global problem occurring with consumption of water at unsustainable rates. As a water threaten and extreme hydrologic event, drought is a hazardous environmental disaster that arises from the long-term insufficiency of water resources influencing nature and live life. In the period of 2002–2021,

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1.4 billion people have been influenced from droughts, caused over 21.000 killing and US$170 billion in economic losses (CRED 2023; United Nations 2024). Drought has been classified into four groups that are meteorological, agricultural, hydrological and socio-economic by many researchers (Deger et al. 2023; Chen et al. 2024; Achite et al. 2024; Leng et al. 2024; Rahmi et al. 2025; Dodangeh et al. 2025; Li et al. 2025; Wang et al. 2025). Across these categories, hydrological droughts are the drought type occurring with declining of surface and ground waters due to long term precipitation deficits. Ghabelnezam et al. (2023) have stated that hydrological drought which takes place due to different activities such as urbanization, industrialization, hydropower generation as a result of water deficiency is considered as most critical drought class. Besides, it has been reported by Athukoralalage et al. (2024) that hydrological drought intensity and frequency can be raised by anthropogenic factors of water withdrawal and allocation for agriculture purposes.

Based on the literature research, there have been many ways in the prediction and as well as the tracking drought events with their effects. Besides, using the drought indices based on a drought type is still very common (Birimbayeva et al. 2024; Wu et al. 2024; Minh et al. 2024; Janardhana and Kikon 2025; Rajput et al. 2025). Among many indices, streamflow drought index (SDI) that has been introduced by Nalbantis and Tsakiris (2009), is a widely preferred index for prediction and assessment of hydrological droughts. Even in many hydrological drought monitoring studies (Simsek 2021; Abro et al. 2022; Yuce et al. 2023; Kartal and Emiroglu 2024; Tuğrul and Hinis 2024; Patidar et al. 2024), SDI is utilized with trend techniques. In fact, during drought monitoring trend assessments of time series produced by drought indexes have the benefits such as detecting drought signs and predicting potential impacts of them.

Although use of indices is very crucial and beneficial for prediction of drought events, an index on its own may not be sufficient to address the potential harms of droughts due to their complex structure making droughts difficult to estimate. Employing drought indices as input variables allows for the analysis of drought characteristics during drought monitoring that are stated by Mishra and Singh (2010). Because drought severity, duration, intensity, and spatial extent influence drought impacts, their consideration enables more effective water resource management, as numerous studies have shown (Shiau 2006; Salvadori and De Michele 2015; Shaw and Chithra 2023; Deger et al. 2023; Yeh et al. 2024) that droughts are multivariate events. Consequently, drought studies have frequently focused on the highly correlated characteristics of severity and duration (Mirabbasi et al. 2012; Tosunoglu and Can 2016; Vazifehkhah et al. 2019; Esit and Yuce 2023; Shaw and Chithra 2023; Deger et al. 2023, 2025). Shiau (2006) has described droughts as stochastic events in which using probabilistic theories is excellent. In the past in order to model a joint distribution, classic multivariate distributions that has many restrictions have been utilized such as in Yue (1999). To overcome those restrictions copula functions that has been introduced by Sklar (1959), have been utilized. With a short definition made by (Shiau 2006) that, copula functions are those that combine univariate distributions to form multivariate distribution functions.

Copula functions have been widely employed to model the joint distribution of drought characteristics across various regions. Copula functions have been employed in several studies (Shiau 2006; Mirabbasi et al. 2012; Abdi et al. 2017; Dehghannik et al. 2021; Hasan and Abdullah 2023; Li et al. 2024b, a; Suo et al. 2024; Bera and Dutta 2024; Meimandi et al. 2024; Wei and Zhao 2024; Aon and Biswas 2024; Kim and Seo 2025; Shao et al. 2025). When it comes to Turkiye research studies of (Tosunoglu and Can 2016; Vazifehkhah et al. 2019; Evkaya et al. 2019; Topçu 2022; Avsaroglu and Gumus 2022; Esit and Yuce 2023; Varol et al. 2023; Gumus et al. 2023; Deger et al. 2023, 2025; Simsek et al. 2024) have been done. However, based on our research in Yeşilırmak Basin which has a critical role for country's development there has not been a drought forecasting study based on copula functions.

In this study, hydrological drought characteristics of severity and duration are modelled by multivariate distribution functions, specifically copulas. To do so, mean monthly streamflow records from 24 stations, obtained from general directorate of state hydraulic works, are first used to determine drought events by employing streamflow drought index (SDI) for a time scale of 3-month. Next, possible trends in SDI-3 time series of all stations are investigated using Mann–Kendall Test, Spearman's Rho Test and Wilcoxon Test to assess the possibility of future droughts. Subsequently, drought severity and duration are calculated using Yevjevich's Run Theory. These series are analyzed by Kendall's τ to assess their suitability for constructing joint distributions. The best marginal distributions of severity-duration series are selected from six distribution types based on goodness of fit tests. Among Gaussian, Student's t, Clayton, Gumbel, Frank, Joe, BB1, BB6, BB7, and BB8 copulas, the joint distribution functions for drought characteristics are determined based on the best copula type selected using tail dependence and goodness of fit tests. Furthermore, conditional probabilities and univariate, bivariate and conditional return periods are calculated using the best copulas.

2 Methodology

2.1 Streamflow drought index (SDI)

Nalbantis and Tsakiris (2009) have developed SDI for prediction of hydrological drought events utilizing monthly streamflow records. In SDI algorithm, the total streamflow that is demonstrated by $X_{i,j}^{k}$ in a given month of *j* and year of *i* depend on the time scale k (1, 3, 6, 9, 12 months) is computed by using equations of 1 and 2 (Paulo et al. 2003; Hong et al. 2015; Deger et al. 2023)

$$X_{i,j}^{k} = \sum_{l=13-k+j}^{12} V_{i-1,l} + \sum_{l=1}^{j} V_{i,l} if j < k \tag{1}$$

$$X_{i,j}^{k} = \sum_{l=j-k+1}^{j} V_{i,l} if j \geq k \tag{2}$$

where $V_{i-1,\ l}$ and $V_{i,l}$ indicates streamflow volumes in the years of *i-1* and *i*, respectively. However the determination procedure starts with fitting an application of Gamma distribution that has been suggested by Nalbantis and Tsakiris (2009). A detailed procedure of Gamma distribution implementation to SDI has been explained in (Deger et al. 2023). In this study SDI-3 series are employed for construction of drought characteristics as SDI-3 has an ability to capture short to medium term drought patterns at the seasonal scale. Compared to other time scales, SDI-1 may have sensitivity in care of short-term fluctuations and longer ones such as SDI-6, SDI-9 can delay detection. From this point, SDI-3 gives a balanced temporal resolution by detecting seasonal dynamics for practical needs in drought monitoring.

2.2 Trend detection tests

To assess the variability of SDI-3 series of all stations within the time, trend behaviors are investigated by trend techniques of Mann–Kendall Test (Kendall 1975), Spearman's Rho Test (Spearman 1904), and Wilcoxon Test (Wilcoxon 1945). The implementation of these tests into time series produced by indexes can be found in (Demirel et al. 2024; Ozocak et al. 2024; Oubadi et al. 2024; Zarei and Mahmoudi 2024; Kartal and Emiroglu 2024; Yaşa and Partal 2024; Aydin et al. 2024; Meskelu et al. 2024; Swain et al. 2024). Possible trends of SDI-3 series are evaluated for the significant level of $\alpha = 0.01$ that corresponds to $Z = \pm 2.576$.

2.3 Drought characteristics and dependence

Mishra and Singh (2010, 2011) stated that drought severity and drought duration are the parameters to characterize the drought fundamentally. Earlier, these two characteristics were defined by Dracup et al. (1980) as follows: drought duration refers to a period between the start and end of a drought event, during which SDI values are below the threshold and drought severity expresses the cumulative summation of SDI values as it is illustrated by Fig. 1. Once SDI values for a selected time scale are known, then the Theory of Runs that was developed by Yevjevich (1967) allows to determine drought severity and duration.

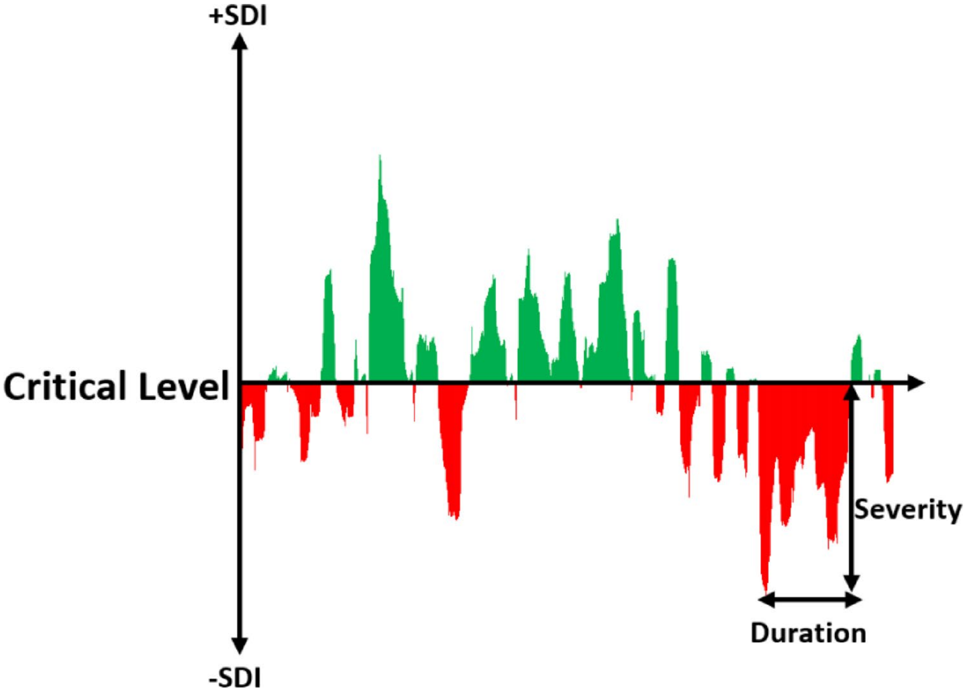


Fig. 1 Graphical representation of drought characteristics

Before construction of multivariate distribution functions, drought severity and drought duration series must have reasonable dependence between each other. To check this suitability, in some studies (Mirabbasi et al. 2012; Khan et al. 2021; Deger et al. 2025; Shao et al. 2025) Kendall's τ that can be computed from Eq. 3 recommended in the literature.

$$\tau = \frac{N_C - N_d}{\left(\frac{1}{2}\right)N(N-1)} \tag{3}$$

where N_c indicates the number of concordant and N_d presents the number of discordant and N shows the sample size.

2.4 Marginal distribution functions and goodness of fit tests

Once reasonable results are obtained from dependence between drought severity and duration, the next step is to construct marginal distributions of each drought characteristics separately, since copula is a multivariate distribution function. To do so, six different marginal distributions which are Normal, Lognormal, Gamma, Weibull, Exponential and Logistics are utilized. Because drought is a serious and hazardous natural disaster, accurate predictions of drought characteristics provide a strong foundation for the predictions of drought by multivariate distributions. Therefore a set of well-known goodness of fit tests which are Kolmogorov–Smirnov (K–S) (Smirnov 1948), Cramer Von Mises (CVM) (Laio 2004), Anderson Darling (AD) (Stephens 1974), Akaike Information Criterion (AIC) (Akaike 1976), Bayesian Information Criterion (BIC) (Stone 1979), Chi-Square (Hamed and Rao 2019) and Maximum Likelihood (MLE) are performed and the best marginal distributions for severity and duration variables are selected based on goodness of fit test results.

2.5 Copulas and tail dependence

Sklar (1959) developed the copula theory to model and analyze dependence structure between random variables of their marginal distribution independently. In the theory, Sklar proposed that for X and Y which are two correlated variables, if $F_{X,\,Y}\,(x,y)$ is a joint distribution function with marginal distributions of $F_X\,(x)$ and $F_Y\,(y)$, then a copula C can be defined as it is shown by Eq. 4.

$$F_{X,Y}\,(x,y) = C\left(F_X\,(x)\,, F_Y\,(y)\right) \tag{4}$$

On the contrary, for any univariate distributions of $F_X\,(x)$ and $F_Y\,(y)$, and any copula of C, the $F_{X,\,Y}\,(x,y)$ that was defined in Eq. 4 indicates a bivariate distribution function with the $F_X\,(x)$ and $F_Y\,(y)$ marginal distributions. If $F_X\,(x)$ and $F_Y\,(y)$ are continuous, then C is unique (Nelsen 2006; Shiau 2006; Esit and Yuce 2023). In case the marginal distributions are treated as continuous with their probability distribution functions (pdf) which are $f_X\,(x)$ and $f_Y\,(y)$, the joint pdf can be expressed as follows.

$$f_{X,Y}\,(x,y) = c\left(F_X\,(x)\,, F_Y\,(y)\right) f_X\,(x)\,, f_Y\,(y) \tag{5}$$

where c shows the density function of C and it is represented by Eq. 6 (Shiau 2006; Tosunoglu and Can 2016; Deger et al. 2025).

$$c\,(u,v) = \frac{\partial^2 C(u,v)}{\partial u \partial v} \tag{6}$$

where u and v are $F_X\,(x)$ and $F_Y\,(y)$ that were represented in Eq. 4.

Copulas have been separated as families which are Archimedean, Elliptical, Extreme and Miscellaneous. Copula types that are used in the study are presented by Table 1.

In extreme events such as droughts, the tail dependence that has been introduced by (Sibuya 1960) is an important concept as it is used to evaluate how extremes of drought severity and duration are connected. In many studies (Mirabbasi et al. 2012; Zhang et al. 2015; Esit and Yuce 2023; Ullah and Akbar 2023; Deger et al. 2023, 2025) tail dependence has been utilized. Xu et al. (2010) have noted that overlooking dependencies may introduce significant uncertainty or errors in quantile predictions, potentially leading to incorrect decisions in hydrological design. In tail dependence assessments, upper tail dependence (λ_u) and lower tail dependence (λ_L) can be computed via from Eq. 7 to Eq. 8 respectively (Nelsen 2006). In the current paper both tail dependency coefficients are calculated however because drought is an extreme hydrological event having damages upper tail dependency is considered. In addition to these two, tail dependencies can be estimated by non-parametric estimators such as $\hat{\lambda}_u^{CFG}$ that was developed by (Frahm et al. 2005) and can be expressed by Eq. 9.

$$\lambda_u = \lim_{t \to 1^-} \frac{1 - 2t + C(t,t)}{1-t} \tag{7}$$

$$\lambda_L = \lim_{t \to 0^+} \frac{C(t,\,t)}{t} \tag{8}$$

$$\hat{\lambda}_u^{CFG} = 2 - 2\exp\left\{\frac{1}{n}\sum_{i=1}^{n}\log\left[\frac{\sqrt{\log\left(\frac{1}{u_1}\right)\log\left(\frac{1}{v_1}\right)}}{\log\left(\frac{1}{\max(u_1,\,v_1)^2}\right)}\right]\right\} \tag{9}$$

Table 1 Selected copula functions and their mathematical representations

Copula type	Function	Support
Gaussian	$C\left(u_1, u_2\right)=\int\limits_{-\infty}^{\phi-1}\left(u_2\right)\int\limits_{-\infty}^{\phi-1}\left(u_1\right)\frac{1}{2\pi(1-p^2)^{\frac{1}{2}}}exp\left\{-\frac{x_1^2+x_2^2-2px_1x_2}{2(1-p^2)}\right\}dx_1x_2$	$x_1, x_2\epsilon R$
Student's t	$C\left(u_1, u_2\right)=\int\limits_{-\infty}^{t_v^{-1}}\left(u_2\right)\int\limits_{-\infty}^{t_v^{-1}}\left(u_1\right)\frac{1}{2\pi(1-p^2)^{\frac{1}{2}}}exp\left\{1+\frac{x_1^2+x_2^2-2px_1x_2}{v(1-p^2)}\right\}^{-(v+2)/2}dx_1x_2$	$x_1, x_2\epsilon R$
Gumbel	$C\left(u_1, u_2\right)=exp\left\{-\left[(-lnu_1)^\theta+(-lnu_2)^\theta\right]^{\frac{1}{\theta}}\right\}$	$\theta\epsilon\left(1, \infty\right)$
Clayton	$C\left(u_1, u_2\right)=\left(u_1^{-\theta}+u_2^{-\theta}-1\right)^{\frac{-1}{\theta}}$	$\theta\epsilon\left(0, \theta\right)$
Frank	$C\left(u_1, u_2\right)=-\frac{1}{\theta}ln\left[1+\frac{\left(e^{-\theta u_1}-1\right)\left(e^{-\theta u_2}-1\right)}{\left(e^{-\theta}-1\right)}\right]$	$\theta\epsilon R$
Joe	$C\left(u_1, u_2\right)=1-\left[\left(1-u_1\right)^\theta+\left(1-u_2\right)^\theta-\left(1-u_1\right)^\theta\left(1-u_2\right)^\theta\right]^{1/\theta}$	$\theta\epsilon\left(1, \infty\right)$
BB1	$C\left(u_1, u_2\right)=\left\{1+\left[\left(u_1^{-\theta}-1\right)^\delta+\left(u_2^{-\theta}-1\right)^\delta\right]^{-1/\theta}\right\}$	$\theta > 0, \delta \geq 1$
BB6	$C\left(u_1, u_2\right)=1-\left(1-exp\left\{-\left[\left(-log\left(1-\left(1-u_1\right)^\theta\right)\right)^\delta+\left(1-log\left(1-\left(1-u_2\right)^\theta\right)\right)^\delta\right]^{\frac{1}{\delta}}\right\}\right)^{\frac{1}{\theta}}$	$\theta \geq 1, \delta \geq 1$
BB7	$C\left(u_1, u_2\right)=1-\left(1-\left[\left(\left(1-u_1\right)^\theta\right)^{-\delta}+\left(1-\left(1-u_2\right)^\theta\right)^{-\delta}-1\right]^{\frac{-1}{\delta}}\right)$	$\theta \geq 1, \delta > 0$
BB8	$C\left(u_1, u_2\right)=\delta^{-1}\left(1-\left\{1-\frac{\left[1-(1-\delta u_1)^\theta\right]\left[1-(1-\delta u_2)^\theta\right]}{1-(1-\delta)^\theta}\right\}^{\frac{1}{\theta}}\right)$	$\theta \geq 1,\ \delta\varepsilon(0,1)$

2.6 Deriving conditional probabilities and univariate, bivariate and conditional return periods by copulas

2.6.1 Conditional probabilities

Conditional probabilities of drought and severity for different conditions that can be derived from a copula based multivariate functions are required for an effective water resources management system. To do so, Shiau (2006) defined these two cases as: first, drought severity distribution given that a drought duration which exceeds a certain threshold d' and second, the conditional drought duration distribution given that a drought severity which exceeds a certain threshold s'. The conditional probabilities of d' and s' can be computed via Eqs. 10 and 11 respectively (Shiau 2006; Azam et al. 2018; Ayantobo et al. 2018; Ahmad et al. 2024).

$$\begin{aligned} \mathrm{P}\left(\mathrm{S} \leq \mathrm{s} \middle| \mathrm{D} \geq \mathrm{d}'\right) &= \frac{\mathrm{P}\left(\mathrm{D} \geq \mathrm{d}', \mathrm{S} \leq \mathrm{s}\right)}{\mathrm{P}(\mathrm{D} \geq \mathrm{d}')} \\ &= \frac{\mathrm{F}\left(\mathrm{s}\right) + \mathrm{F}\left(\mathrm{d}', \mathrm{s}\right)}{1 - \mathrm{F}\left(\mathrm{d}'\right)} \\ &= \frac{\mathrm{F_s}\left(\mathrm{s}\right) - \mathrm{C}\left(\mathrm{F_D}\left(\mathrm{d}'\right), \mathrm{F_S}\left(\mathrm{s}\right)\right)}{1 - \mathrm{F_D}\left(\mathrm{d}'\right)} \end{aligned} \tag{10}$$

$$\begin{aligned} \mathrm{P}\left(\mathrm{D} \leq \mathrm{d} | \mathrm{S} \geq \mathrm{s}'\right) &= \frac{\mathrm{P}\left(\mathrm{D} \leq \mathrm{d}, \mathrm{S} \geq \mathrm{s}'\right)}{\mathrm{P}(\mathrm{S} \geq \mathrm{s}')} \\ &= \frac{\mathrm{F}\left(\mathrm{d}\right) - \mathrm{F}\left(\mathrm{d}, \mathrm{s}'\right)}{1 - \mathrm{F}\left(\mathrm{s}'\right)} \\ &= \frac{\mathrm{F_D}\left(\mathrm{d}\right) - \mathrm{C}\left(\mathrm{F_D}\left(\mathrm{d}\right), \mathrm{F_S}\left(\mathrm{s}'\right)\right)}{1 - \mathrm{F_S}\left(\mathrm{s}'\right)} \end{aligned} \tag{11}$$

2.6.2 Univariate, bivariate and conditional return periods

Given that extreme events may occur repetitively, addressing the return periods of hazardous events such as drought provides a precaution against potential harms to the environment and economy. From this point, return periods of drought severity and duration based on certain cases give an idea to water resources managers and planners for sustainability. Shiau (2006) has defined univariate return period of drought duration ($\mathrm{T_D}$) and univariate return period of drought severity ($\mathrm{T_S}$) that are greater or equal to a certain values. These two values of $\mathrm{T_D}$ and $\mathrm{T_S}$ can be determined by Eq. 12 and Eq. 13 respectively (Shiau 2006). In those equations L presents the drought interarrival time that shows the period which is between the beginning of two consecutive droughts and E(L) is indicates the expected interarrival time value. The detailed derivation of these formulas can be seen in (Yue and Rasmussen 2002).

$$\mathrm{T_D} = \frac{\mathrm{E(L)}}{1 - \mathrm{F_D(d)}} \tag{12}$$

$$T_S = \frac{E(L)}{1 - F_S(s)} \tag{13}$$

In addition to univariate return periods Shiau (2006) has categorized the bivariate return periods of drought events in two cases: first case includes T_{DS} which is for D≥d and S≥s while the second case is and T'_{DS} that is for D≥d or S≥s. T_{DS} and T'_{DS} can be computed by Eqs. 14 and 15 respectively.

$$\begin{aligned} \mathrm{T_{DS}} &= \frac{\mathrm{E(L)}}{\mathrm{P(D \ \geq \ d, \ S \ \geq \ s)}} \\ &= \frac{\mathrm{E(L)}}{1 \text{ - } \mathrm{F_D}\,(\mathrm{d}) \text{ - } \mathrm{F_S}\,(\mathrm{s}) + \mathrm{F_{DS}(d,\ s)}} \\ &= \frac{\mathrm{E(L)}}{1 \text{ - } \mathrm{F_D}\,(\mathrm{d}) \text{ - } \mathrm{F_S}\,(\mathrm{s}) + \mathrm{C(F_D\,(d)\,,F_S\,(s))}} \end{aligned} \tag{14}$$

$$\begin{aligned} \mathrm{T'_{DS}} &= \frac{\mathrm{E(L)}}{\mathrm{P(D \geq dorS \geq s)}} \\ &= \frac{\mathrm{E(L)}}{1 - \mathrm{F_{DS}(d,s)}} \\ &= \frac{\mathrm{E(L)}}{1 - \mathrm{C(F_D\,(d)\,, F_S\,(s))}} \end{aligned} \tag{15}$$

where $F_S(s)$ and $F_D(d)$ indicate the cumulative distribution functions of univariate drought severity, and duration respectively, and C states an any type of copula.

Similar to univariate and bivariate return periods, return periods can be expressed as conditional cases. The return period of drought duration given drought severity exceeding a certain threshold and the return period of drought severity given drought duration exceeding a certain threshold can be defined with Eqs. 16 and17 respectively (Shiau 2006).

$$\begin{aligned} \mathrm{T_{D|S \geq s}} &= \frac{T_S}{\mathrm{P(D \ \geq \ d, \ S \ \geq \ s)}} \\ &= \frac{\mathrm{E(L)}}{[1 \text{ - } \mathrm{F_S\,(s)}]\,[1 \text{ - } \mathrm{F_D}\,(\mathrm{d}) \text{ - } \mathrm{F_S}\,(\mathrm{s}) + \mathrm{C(F_D\,(d)\,,F_S\,(s))}]} \end{aligned} \tag{16}$$

$$\begin{aligned} \mathrm{T_{S|D \geq d}} &= \frac{\mathrm{T_D}}{\mathrm{P(D \ \geq \ d, \ S \ \geq \ s)}} \\ &= \frac{\mathrm{E(L)}}{[1 \text{ - } \mathrm{F_D\,(d)}]\,[1 \text{ - } \mathrm{F_D}\,(\mathrm{d}) \text{ - } \mathrm{F_S}\,(\mathrm{s}) + \mathrm{C(F_D\,(d)\,,F_S\,(s))}]} \end{aligned} \tag{17}$$

where $\mathrm{T}_{\mathrm{D|S \geq s}}$ represents the conditional return period for D given $\mathrm{S} \geq \mathrm{s}$ and $\mathrm{T}_{\mathrm{S|D \geq d}}$ describes conditional return period for S given $\mathrm{D} \geq \mathrm{d}$.

3 Data and study area

Yeşilırmak Basin which covers the 5% of Turkey's terrain is found with the coordinates of 40° 38′ 54″ North latitude 35° 49′ 52″ East longitude, in North Anatolian part. The basin pours its water to Black Sea via -519 km long- Yeşilırmak River that has 3 main branches: Tersakan Stream, Kelkit Stream and Çekerek Stream. In coastal regions where summers are hot and winters are warm and wet, the impacts of Black Sea climate is evident, while inland areas experience colder winters with snow and cool summers due to high mountains (SYGM 2015). It has been stated by (Serencam 2019) that average annual precipitation is 646 mm in the basin and average annual flow is 5.80 km^3. The basins of Euphrates-Tigris, Seyhan, Kızılırmak, West and East Black Sea are the neighbors of the basin. The basin area consists of 19% grass areas, 39% cultivated areas and 39% forestlands. It has been stated by (Katipoğlu 2023) that the basin that has more dam lakes than other basins in Turkey has a critical significance for hydroelectric energy generation, drinking, irrigation, industrial water. In the study, mean monthly streamflow data of 24 stations which have taken from General Directorate of State Hydraulic Works (DSI) of Turkey have been used and locations as well as the map of the basin have been given in Fig. 2.

Statistical information of used stations has been given in Table 2. There's a wide variation in flow rates across stations. The highest maximum flow was recorded at station E14A002 (765.26 m^3/s). Many stations recorded minimum flows of 0.00 m^3/s, suggesting seasonal or intermittent flow. Mean flows vary significantly, from as low as 0.11 m^3/s to as high as 144.62 m^3/s. Standard deviations indicate high variability in flow rates at some stations. Coefficient of variance shows the relative variability of flows Kurtosis values range from about 1.67 to 27.98, indicating different flow distribution patterns. Skewness coefficients are all positive, suggesting right-skewed distributions.

4 Results and discussion

4.1 Investigation of SDI values

This study first employs the SDI algorithm to predict SDI-3 and potential drought events in the basin. To explore the behaviors of SDI-3 values during the time interval in one dimension (based on obtained SDI values), some examples of constructed charts belong to different stations are given by Fig. 3. As it can be seen from Fig. 3, many drought events (red colored) are observed in those stations. Even near today serious drought events with long durations can be detected from the charts. From this point and all obtained charts of stations, the results have shown that the basin has experienced with hydrological drought with different time intervals.

4.2 Trend detection in SDI-3

In order to understand possibility of future droughts in the basin, Mann–Kendall test, Spearman's Rho Test and Wilcoxon Test have been applied to SDI-3 series of all stations considering significant level of α = 0.01 that corresponds

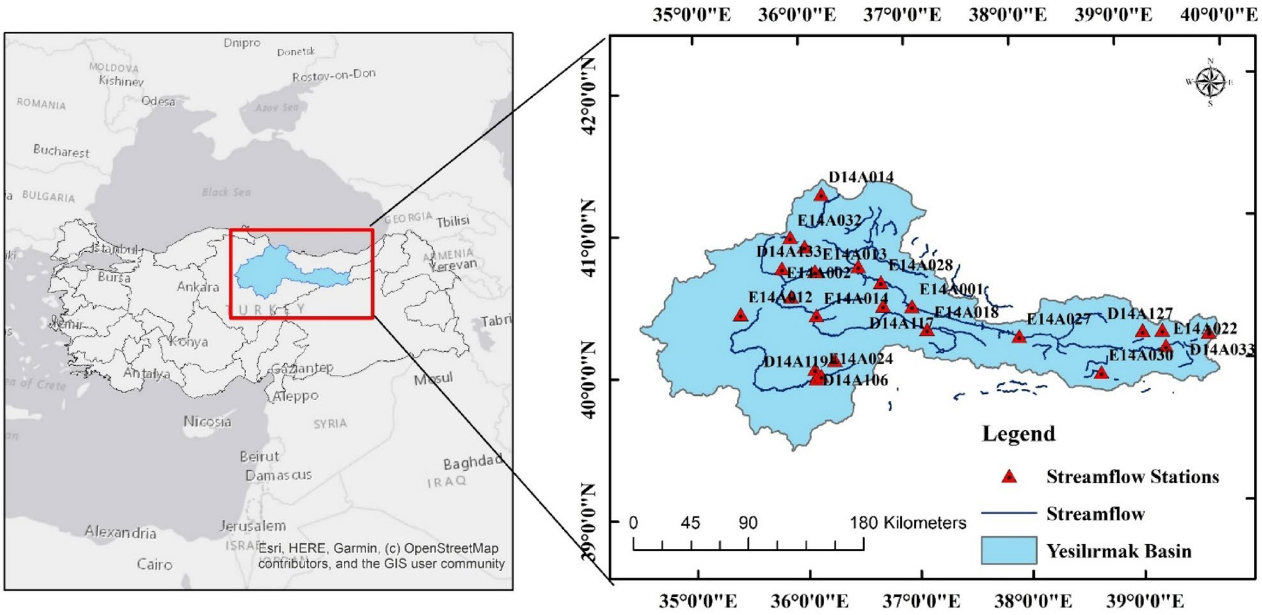


Fig. 2 The location of selected streamflow station in Yeşilırmak Basin

Table 2 Information of used stations in Yeşilırmak Basin

Station	Latitude (N)	Longitude (E)	Elevation (m)	Data interval	Minimum flow (m^3/s)	Maximum flow (m^3/s)	Mean flow (m^3/s)	Std. deviation	Coefficient of variance	Kurtosis	Coefficient of Skewness
D14A011	40°55′13"	36°1′14"	862	1999–2019	0.00	10.32	1.49	1.97	1.32	2.41	1.64
D14A014	41°17′10"	36°11′13"	140	1964–2019	0.00	8.93	1.42	1.48	1.04	3.29	1.73
D14A024	40°34′7"	35°53′9"	502	1976–2019	0.00	6.76	0.87	0.99	1.13	5.22	2.04
D14A033	40°11′27"	39°42′39"	1558	1984–2019	0.01	4.87	0.40	0.53	1.32	16.51	3.29
D14A062	40°6′56"	36°16′26"	1232	1968–2019	0.00	1.80	0.11	0.15	1.44	27.98	4.03
D14A106	40°3′22"	36°5′33"	1095	1979–2017	0.00	2.91	0.39	0.49	1.25	5.47	2.12
D14A117	40°29′21"	36°43′56"	734	1985–2018	0.04	3.48	0.58	0.63	1.09	2.81	1.72
D14A119	39°59′23"	36°6′5"	1050	1988–2015	0.00	2.53	0.28	0.38	1.33	7.80	2.49
D14A125	40°13′39"	39°17′13"	1610	1988–2013	0.00	4.00	0.32	0.49	1.52	10.99	2.67
D14A126	40°13′56"	39°6′28"	1500	1990–2017	0.00	2.93	0.33	0.49	1.50	6.18	2.38
D14A127	40°13′47"	39°6′28"	1497	1989–2017	0.00	12.68	1.48	2.36	1.60	5.86	2.37
D14A133	40°46′10"	35°48′18"	600	1996–2019	0.00	2.30	0.25	0.31	1.26	7.73	2.33
E14A001	40°28′42"	36°59′56"	375	1938–2011	0.00	547.77	70.20	79.27	1.13	5.02	2.17
E14A002	40°46′18"	36°30′45"	190	1962–2019	13.47	765.26	144.62	112.53	0.78	4.58	1.91
E14A012	40°27′6"	35°25′3"	530	1954–2019	0.00	59.12	6.08	7.57	1.25	9.38	2.66
E14A013	40°44′40"	36°6′43"	301	1955–2012	2.47	350.17	58.42	52.26	0.89	4.01	1.82
E14A014	40°25′59"	36°6′56"	510	1955–2019	0.00	139.05	21.52	18.26	0.85	6.27	2.05
E14A018	40°18′42"	37°7′43"	820	1965–2019	0.00	111.38	18.10	20.33	1.12	2.58	1.75
E14A022	40°6′55"	39°18′42"	1350	1969–2019	0.08	70.70	7.61	10.34	1.36	5.12	2.18
E14A024	40°0′29"	36°8′47"	1040	1969–2019	0.00	29.07	3.67	4.55	1.24	6.32	2.36
E14A027	40°14′17"	37°58′21"	690	1982–2019	0.59	319.60	40.77	47.84	1.17	9.17	2.61
E14A028	40°39′15"	36°43′7"	275	1990–2013	19.85	340.13	81.47	43.07	0.53	5.98	1.87
E14A030	39°57′17"	38°42′28"	1475	1998–2019	0.59	16.28	2.87	3.33	1.16	3.88	2.14
E14A032	40°59′13"	35°53′15"	758	2000–2018	0.05	11.03	2.29	1.95	0.85	1.67	1.24

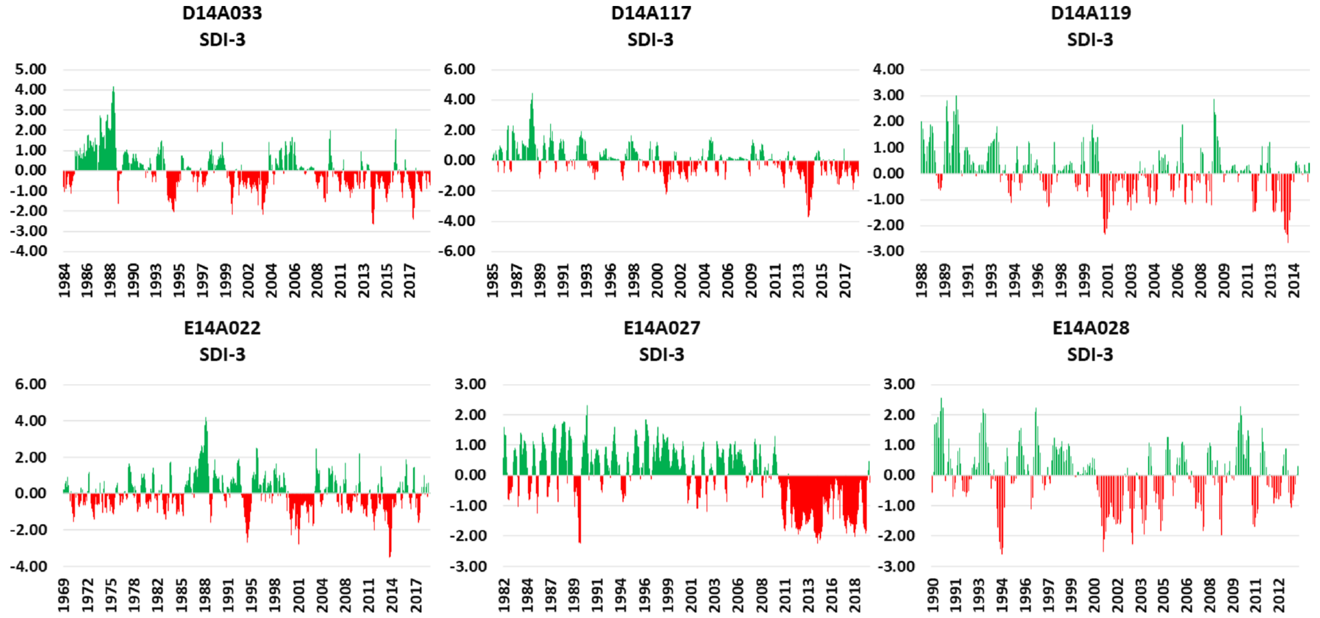


Fig. 3 Investigation of SDI-3 time series of some stations in Yeşilırmak Basin

to $Z = \pm 2.576$. Overall results are illustrated in Fig. 4 on the basin as well as for tracking the distribution of different trend types. Mann–Kendall results are indicated that 15 stations have shown a significant decreasing trend while 7 stations shown a decreasing trend but not significant. E14A001 station that is very near to middle has shown a significant increasing trend while D14A014 station has been also detected with increasing trend but non-significant. In

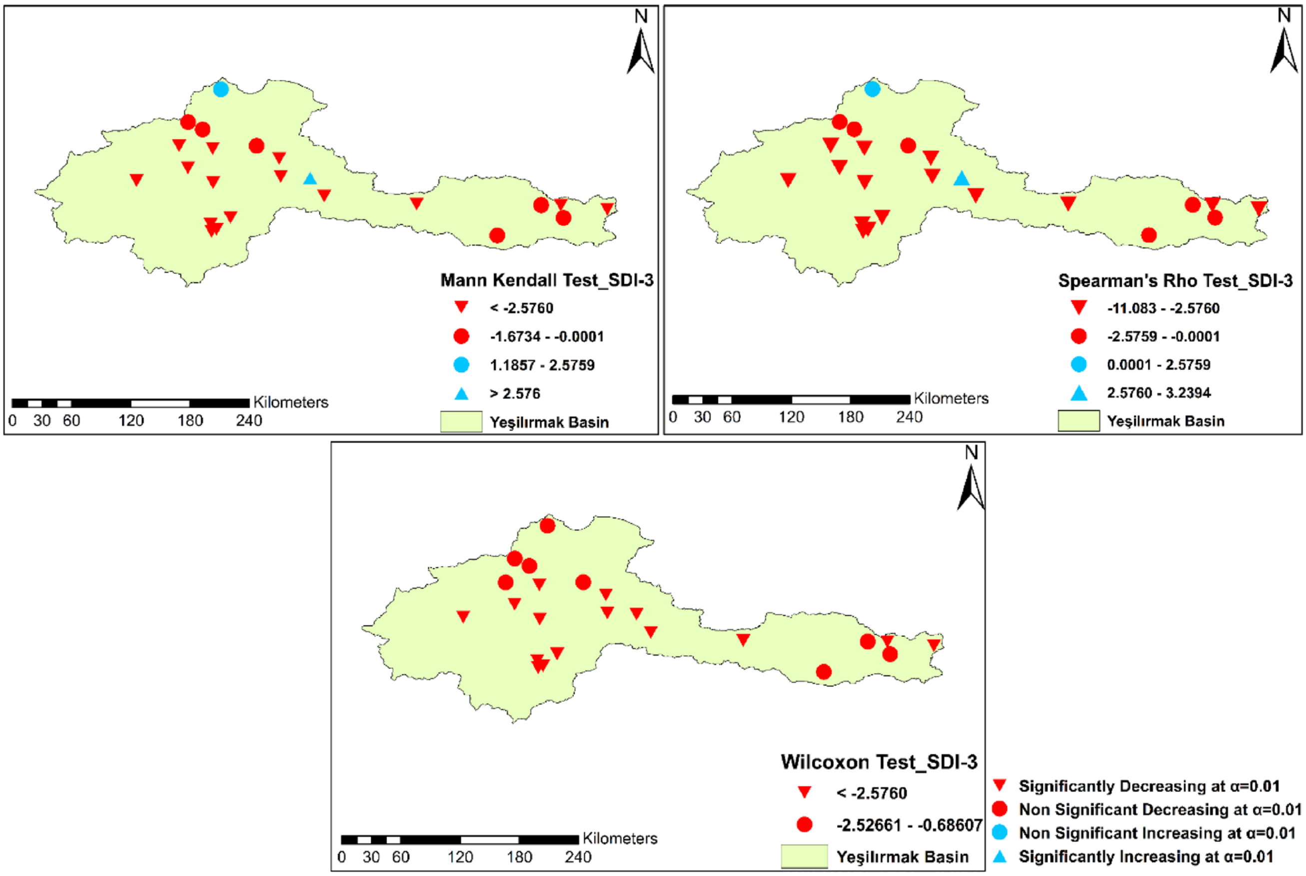


Fig. 4 Trend results of Mann–Kendall, Spearman's Rho and Wilcoxon Tests in SDI-3

Spearman's Rho test, trend results of SDI-3 have shown that 15 stations have been observed with significant decreasing trends while 7 stations show non-significant decreasing trends. Similar to Mann–Kendall test E14A001 station have been noted with significant increasing trend and D14A014 station with non-significant increasing trend. When it comes to Wilcoxon Test, the results have indicated that 15 stations have shown significant decreasing trend behavior while 9 stations were noted with non-significant decreasing trends. Unlike the Mann–Kendall and Spearman's Rho Tests, no increasing trend neither significant nor non-significant has been observed. As common, 14 stations (D14A024, D14A033, D14A062, D14A106, D14A117, D14A119, D14A125, D14A126, D14A127, D14A133, E14A001, E14A002, E14A012, E14A013, E14A014, E14A018, E14A022, E14A024, E14A027, E14A028) have shown significant decreasing trend in 3 tests. Besides, the stations of D14A011, D14A126, D14A127, E14A002, E14A022, E14A030, E14A032 have shown non-significant decreasing trends in 3 tests. From these results that are illustrated in Fig. 4, it has been noted that there has been a prevalence of significant trends in Yeşilırmak Basin with SDI-3. The results indicates that SDI values are continuously decreasing within the time in most of the stations.

4.3 Investigation of drought characteristics' dependence

Via (Yevjevich 1967)'s Run Theory and SDI-3 values, drought severity and duration series are obtained for all stations. Secondly, the dependence between these two characteristics are investigated by Kendall's τ correlation coefficient. All obtained τ values have been presented via Fig. 5 as schematically on the basin map. As it can be seen from Fig. 5, Kendall's τ values have been ranged between 0.719 and 0.907 which indicates a strong positive correlation. Therefore, the severity and duration series are accepted as convenient for obtaining joint distribution functions.

4.4 Marginal distribution analysis of drought characteristics

After obtaining good results in dependence analysis, marginal distributions of drought characteristics are determined separately. To do so, the best fitted distribution are

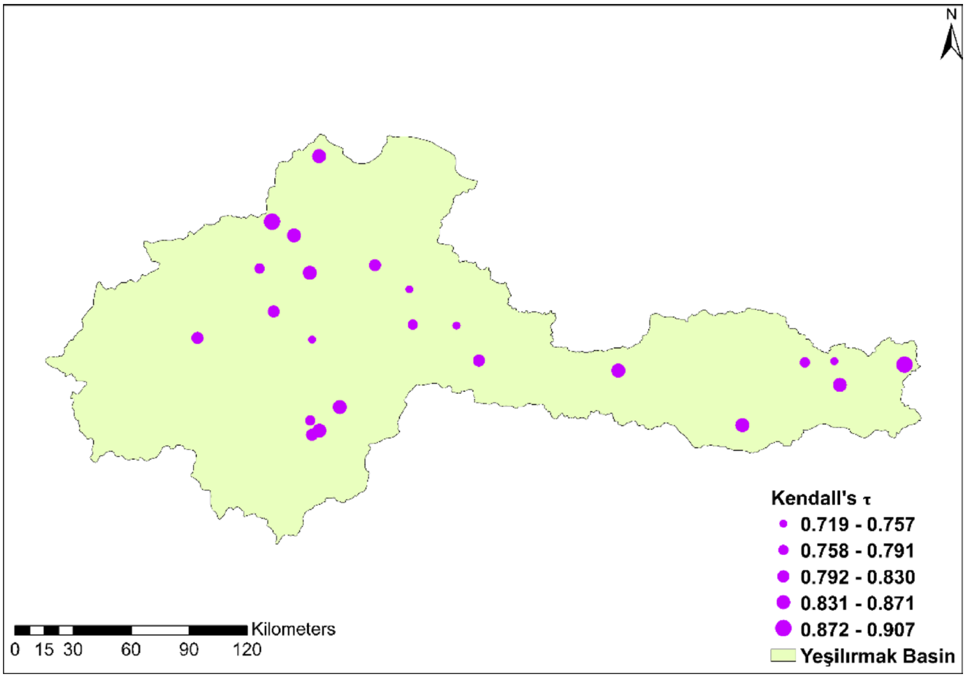


Fig. 5 Dependence of severity and duration based on Kendall's τ

Table 3 Selection of the best marginal distributions of drought characteristics in E14A022 station

	K–S	CVM	AD	AIC	BIC	CS	MLE	Parameters
E14A022								
Severity								
Weibull	0.0545	0.0279	0.2379	241.5375	245.5155	3.5304	− 118.7688	[0.6220,2.8864]
Duration								
Lognormal	0.17937	0.18413	1.46150	293.55281	297.53077	23.17918	− 144.77640	[1.2746,0.9875]

selected for severity and duration among the distributions of Normal, Lognormal, Gamma, Weibull, Exponential and Logistics based on the results obtained by goodness of fit tests. All results of best fit distribution with their results are given in Supplementary. Nevertheless, an example of best fits of drought characteristics in E14A022 station are given in Table 3. For either severity or duration series, a distribution are selected based on the smallest statistics in a goodness of fit test. A distribution then are selected the best fit distribution when it gets the superiority among all distribution in terms of numbers of goodness of fit tests. Beside of testing as numerically, selected distributions of drought series are illustrated as graphically. A typical distribution fits for E14A022 station are illustrated in Fig. 6. The graphs show that how Weibull fits well in severity, Lognormal in duration.

Based on the results in Supplementary, a summarize of best fits of marginal distributions are given in Fig. 7. Figure 7 demonstrates the superior fit of the Lognormal distribution for drought duration (selected 15 times) and the Weibull distribution for drought severity (selected 11 times). While the Weibull distribution was the second-best fit for duration (4 times), the Lognormal distribution held a similar position for severity (9 times), indicating a close competition. Notably, neither the Normal nor the Logistic distribution was selected as the best fit in any instance. The Gamma and Exponential distributions exhibited comparable performance.

4.5 Copula and tail dependence assessments

From the best marginal distributions of drought characteristics, 10 types of copula functions are used for construction of multivariate distribution functions. To do so, copulas are tested firstly by tail dependence criteria since it plays a critical role in hydrologic extreme events' predictions and secondly by goodness of fit tests of MLE, AIC and BIC. As an example, a best copula for E14A022 station are selected as follows: At first non-parametric tail dependence $\widehat{\lambda}_u^{\mathrm{CFG}}$ which is equal to 0.829 has been compared with the closest upper tail dependence values of copula classes that also allows to eliminate Gaussian, Clayton, Frank and BB8 copulas since they have 0 in upper tail. From this point, Gumbel copula and BB6 are seen as the candidate copulas for being the best based on Table 4. In such cases, the goodness of fit test results has been considered as second check. As it can

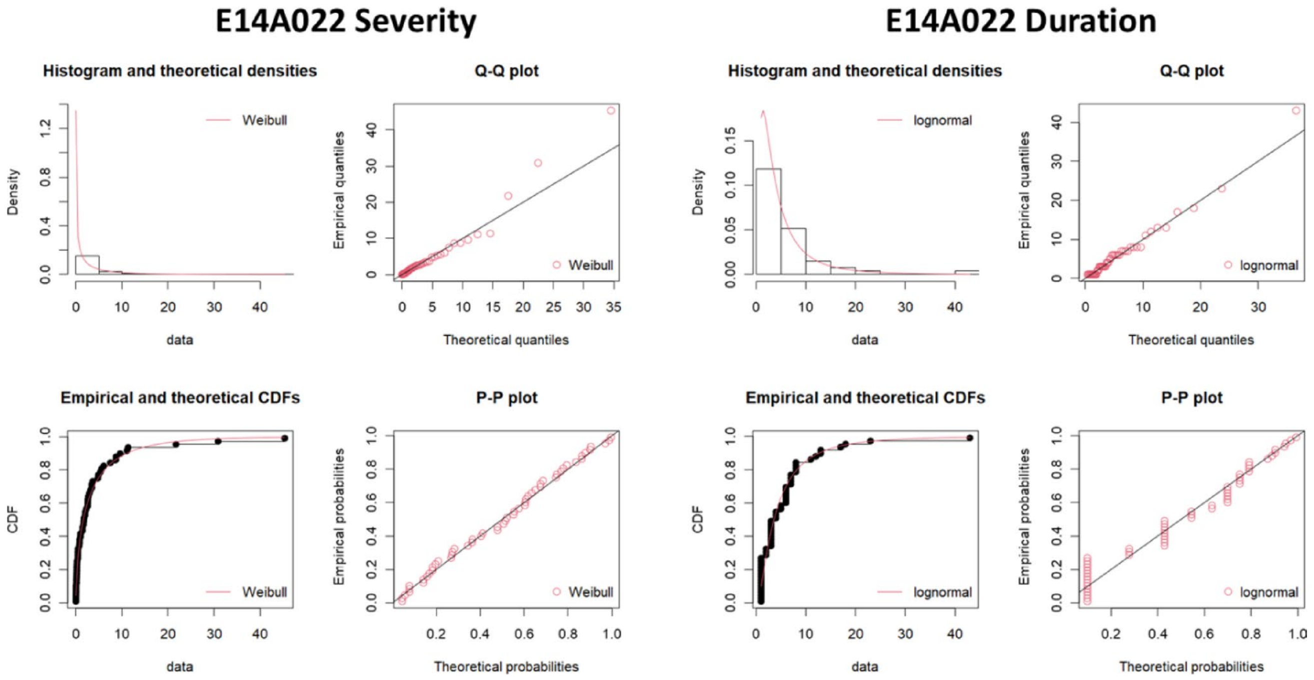


Fig. 6 The graphical illustration of the best fit distributions of drought characteristics in E14A022 station

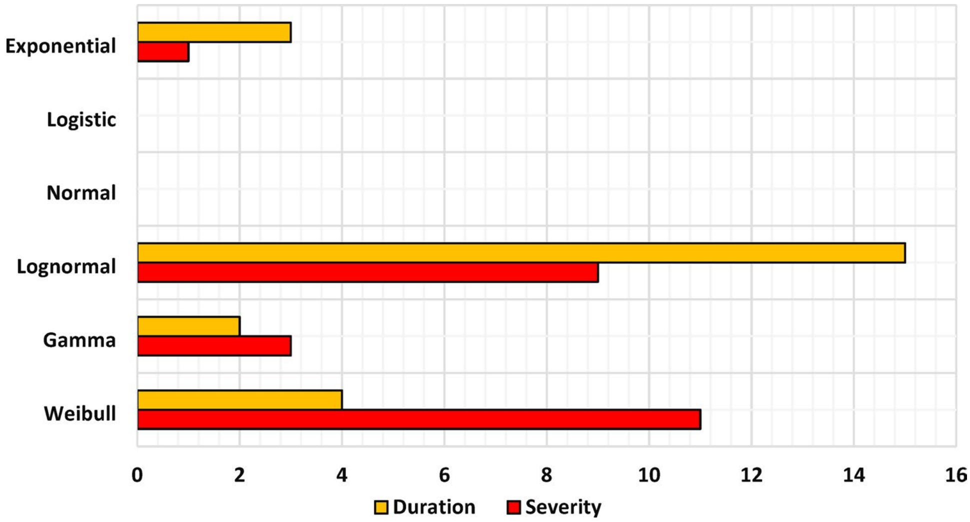


Fig. 7 The distribution of the best fits of drought characteristics in Yeşilırmak Basin

be seen, Gumbel copula has superior to BB6 copula since it has smallest values in AIC and BIC tests and bigger in MLE. Therefore for this sample, Gumbel copula has been selected and shown with bold font in Table 4. Overall, all selected copulas including its parameters and test results have been given Supplementary. Based on the overall results, Gumbel copula has the superiority in Yeşilırmak Basin as it has been detected as the best copula by 23 times while only in E14A013 station BB6 copula has been selected as the best copula type.

In addition to selecting best copulas via criteria stated above the best copulas have been tested via a visual test. To do so, 10.000 random pairs have been produced and simulated. Some examples of the test have been given with Fig. 8. From the figure, it is very clear that observed findings by selected copulas are on the bounds of the pairs. In all 24 stations same results have been obtained for selected copulas indicating that the selected copulas are appropriate as multivariate functions for further analysis.

4.6 Conditional distributions of drought characteristics

Via the best copulas, the conditional probabilities of drought cases which are helpful for water resources management have been studied. At first, the conditional probability of

Table 4 Selection of the best copula type for E14A022 station

Copula family	Goodness of fit tests			Parameters		Tail dependence		Non-parametric tail dependence (λ_u^{CFG})	Kendall's τ
	MLE	AIC	BIC	Par. 1	Par. 2	Lower tail	Upper tail		
Gaussian copula	64.331	−126.661	−124.672	0.954	0.000	0.000	0.000	0.829	0.834
Student's t copula	63.982	−123.964	−119.986	0.953	30.000	0.392	0.392		
Clayton copula	44.298	−86.597	−84.608	3.994	0.000	0.841	0.000		
Gumbel copula	**62.091**	**−122.182**	**−120.193**	**4.612**	**0.000**	**0.000**	**0.838**		
Frank copula	58.696	−115.392	−113.403	17.697	0.000	0.000	0.000		
Joe copula	55.206	−108.411	−106.422	5.897	0.000	0.000	0.875		
BB1 copula	62.777	−121.553	−117.575	0.332	4.022	0.595	0.812		
BB6 copula	62.088	−120.177	−116.199	1.001	4.609	0.000	0.838		
BB7 copula	60.212	−116.423	−112.445	5.000	2.861	0.785	0.851		
BB8 copula	57.331	−110.663	−106.685	6.000	0.975	0.000	0.000		

Bold font shows selected copula

drought duration given that drought severity exceeds a certain value which s′ has been considered for all stations. For example, the case indicating the conditional probability for drought duration less than 10 months given a specific severity exceeding 6 have been computed as 0.351 in D14A011, 0.126 in D14A033, 0.506 in D14A062, 0.313 in E14A022 and so on. The same probability has been determined with the range of 0.126–0.547 in all the basin. Secondly, conditional probability of drought severity given drought duration exceeding a certain value *d′* is has been investigated for all stations. As an example, the conditional probability for drought severity less than 10 given drought duration exceeding 6 months have been determined as 0.643 in D14A011, 0.605 in D14A033, 0.695 in D14A062, 0.620 in E14A022. The same probability has been computed with the range of 0.527–0.873 in all the basin. Graphical results of these probabilities have been given with Fig. 9. It is very clear that for the selected case either in *d′* condition or s′ condition, the stations with higher probabilities states more risk of drought. Addition to this point view, since the probability range of selected case in *d′* condition gave higher results than selected case of s′ condition, then it contains more risk of drought.

4.7 Univariate, bivariate and conditional return periods

In the current paper, return periods of drought characteristics are modelled as univariate, bivariate and conditional. Based on the methodology explained in 2.6.2, firstly univariate drought duration and severity are determined considering the return periods of 10, 20, 50, 100, 200 and 500-years. Then after, bivariate T_{DS} (AND) and T'_{DS} (OR) cases which are corresponding to determined drought duration (D) and drought severity (S) in selected return periods have been calculated. Some numerical results of the stations are tabulated in Table 5. In addition to this, T_{DS} (AND) and T'_{DS} (OR) return period relationships are graphically illustrated in Fig. 10. For example, in D14A127 station for 200-year return period univariate drought duration (D) is 20.95 months while the severity (S) is 24.17. T_{DS} describing the return period in which two variables exceed a certain value corresponding to D and S of this case has been determined as 253.40 years while T'_{DS} that represents the return period in which one of two variables exceeds a certain value corresponding to D and S of this case is 165.19 years. From the definition of both bivariate return periods, it is very natural that the return periods of T_{DS} (AND) are observed as longer than the T'_{DS} (AND) in all stations. Besides, bivariate return periods provide more information about the likelihood of two variables (D and S) occurring or not occurring at the same time. This highlights the utility of bivariate return periods, derived using copulas, in providing a more comprehensive understanding of the joint probability of drought severity (S) and duration (D) exceeding critical levels, information not readily available from univariate analyses. In order to assess the distribution of findings of univariate and bivariate return periods in terms of risks for the basin, the values obtained are spatially analyzed. To do so, using inverse distance weighting (IDW) module of a licensed version of ArcGIS software, spatial interpolation technique are employed. The distribution results are given with Fig. 11. With this manner, numerical results and as well as the variability of D, S, T_{DS} (AND), T'_{DS} (OR) considering spatial and return period can be easily investigated as they also can be compared easily. As it can be seen, 10-year return period drought duration are observed as longer in west, north the intersection of west and north and a small part of east, with a total range of 8.09–16.23 months. Severity with a range of 6.71–13.6 is recorded as higher in north, some parts of middle, some part of east and west. T_{DS} with a distribution range of 11.34–13.55 years are detected as higher in the intersection

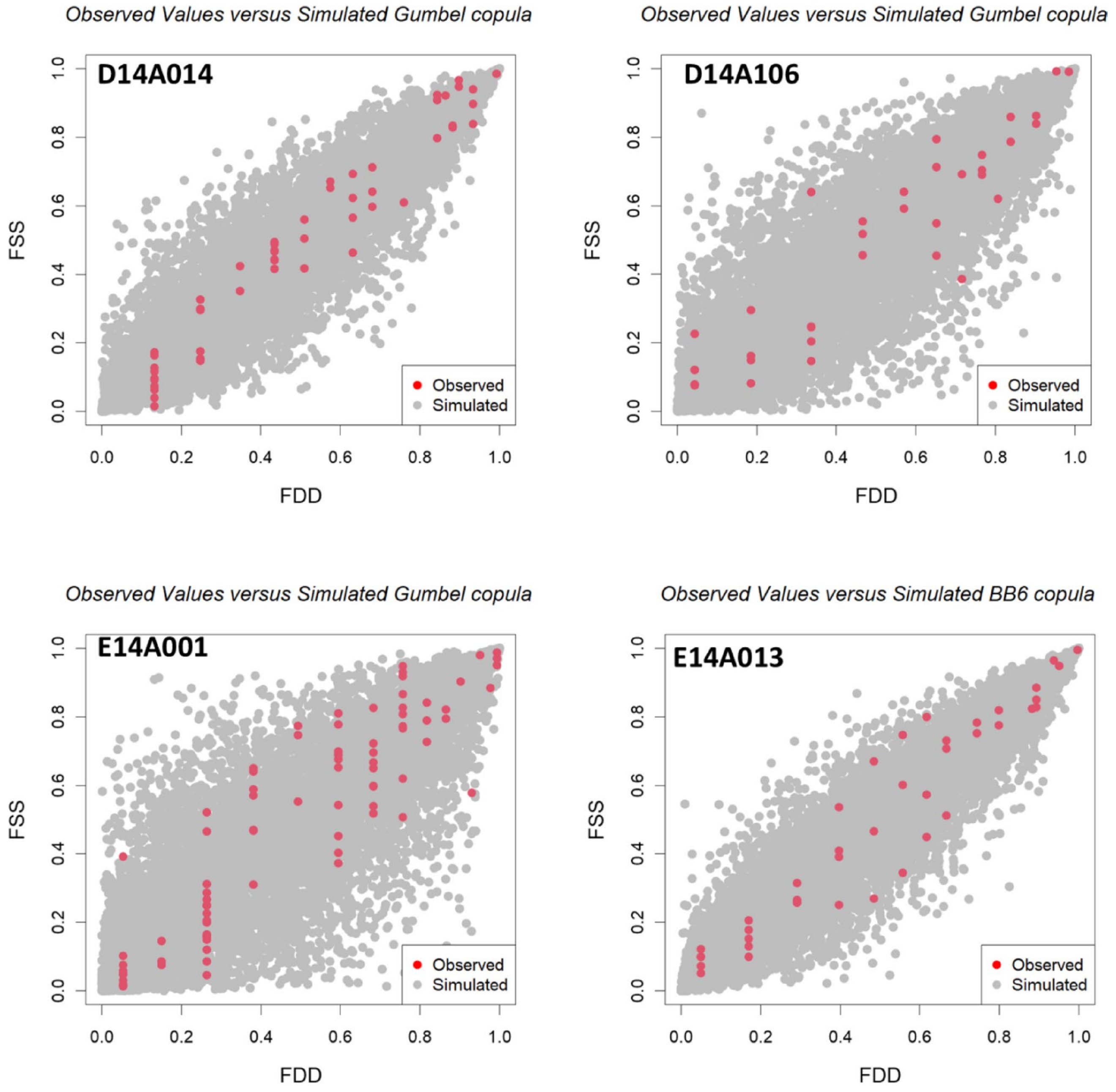


Fig. 8 Testing the copulas via a visual test

of middle and east, some parts of east while lighter in the intersection of west and north and in considerable amount of west. T'_{DS} results (7.92–8.94 years) are indicated that a significant part of basin in the range of 8.43–8.94 years. In 20-year return period, drought duration is recorded with a total range of 10.59–22.43 months and it is noted as longer in west, north, some parts of middle and small portions of east. Drought severity with a distribution range of 9.77–24.57 is observed more severe in west and north sides with range of 17.28–24.57, even some parts of east and middle regions can be linked to this range. T_{DS} with a distribution range of 22.77–27.41 years is detected with higher values in the intersection of middle and east and some parts of east. T'_{DS} (15.74–17.83 years) is recorded with higher values in all the west side, nearly all north side and west side. In 50-year return period, drought duration is recorded with the distribution range of 14.39–36.17 months and as it is seen from the figure, the higher values is observed in west side and small portions of east side. Univariate drought severity (13.2–55.59) is also detected as higher in west side. T_{DS} whose distribution range has become 57.04–68.99 years is observed as higher in the intersection of middle and east and

Fig. 9 The conditional distribution cases of s′(left) and d′(right) for drought characteristics

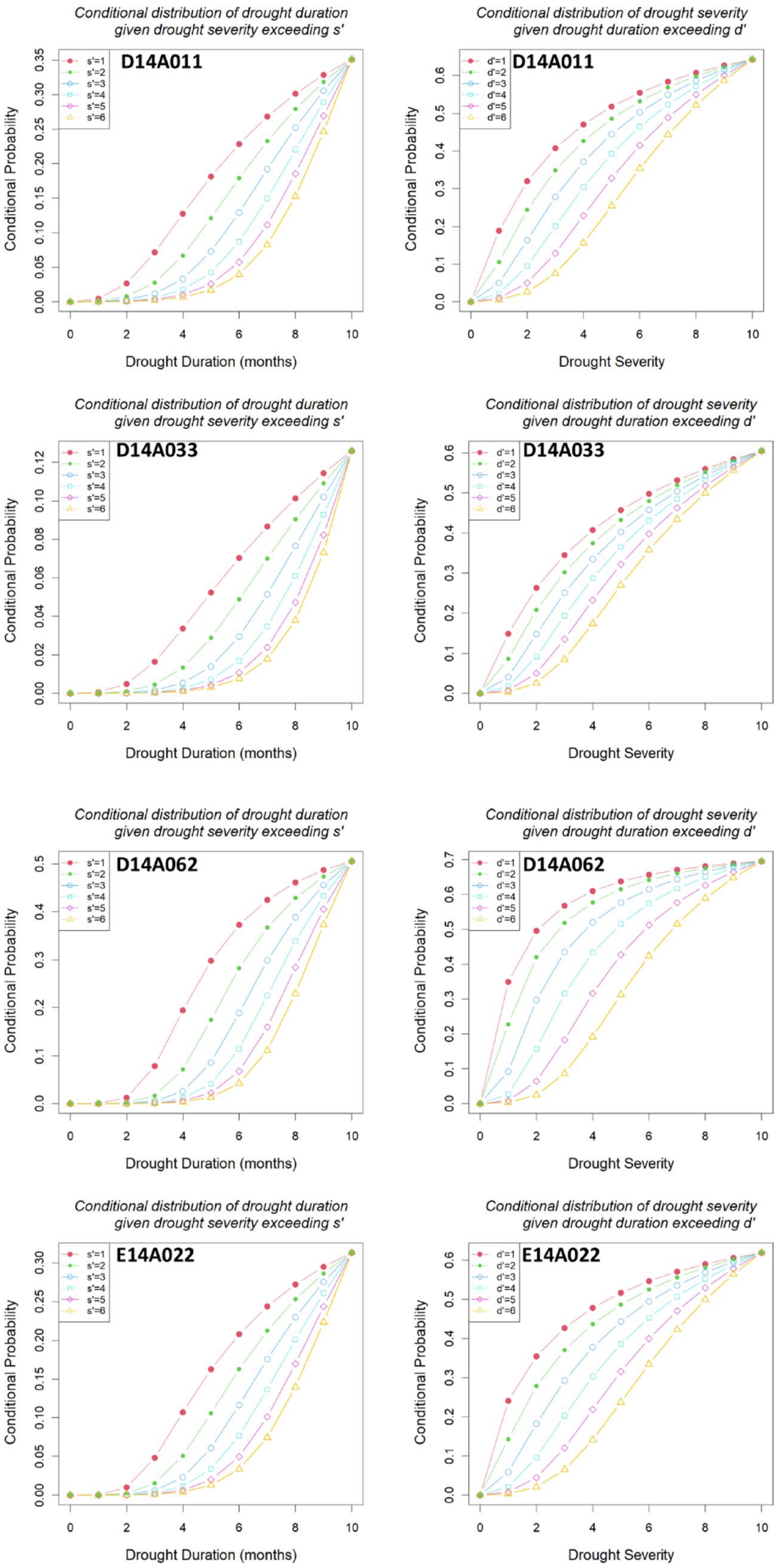


Table 5 Univariate and bivariate return periods of some stations

Station	Year	D	S	T_{DS} (AND)	T'_{DS} (OR)
D14A126	10	10.53	9.58	13.57	7.92
	20	12.41	12.53	27.46	15.73
	50	14.68	16.45	69.15	39.16
	100	16.27	19.44	138.61	78.21
	200	17.78	22.42	277.55	156.32
	500	19.67	26.39	694.36	390.65
D14A127	10	11.06	10.07	12.46	8.35
	20	13.46	13.32	25.14	16.60
	50	16.51	17.63	63.18	41.37
	100	18.75	20.90	126.59	82.64
	200	20.95	24.17	253.40	165.19
	500	23.81	28.51	633.82	412.84
E14A001	10	11.12	10.49	13.37	7.99
	20	13.12	13.57	27.03	15.87
	50	15.53	17.64	68.03	39.52
	100	17.22	20.71	136.37	78.95
	200	18.82	23.79	273.04	157.79
	500	20.82	27.86	683.04	394.33
E14A022	10	13.16	11.55	11.81	8.67
	20	18.74	17.45	23.74	17.28
	50	27.93	26.58	59.55	43.09
	100	36.47	34.40	119.23	86.11
	200	46.58	42.97	238.58	172.16
	500	62.66	55.37	596.65	430.30

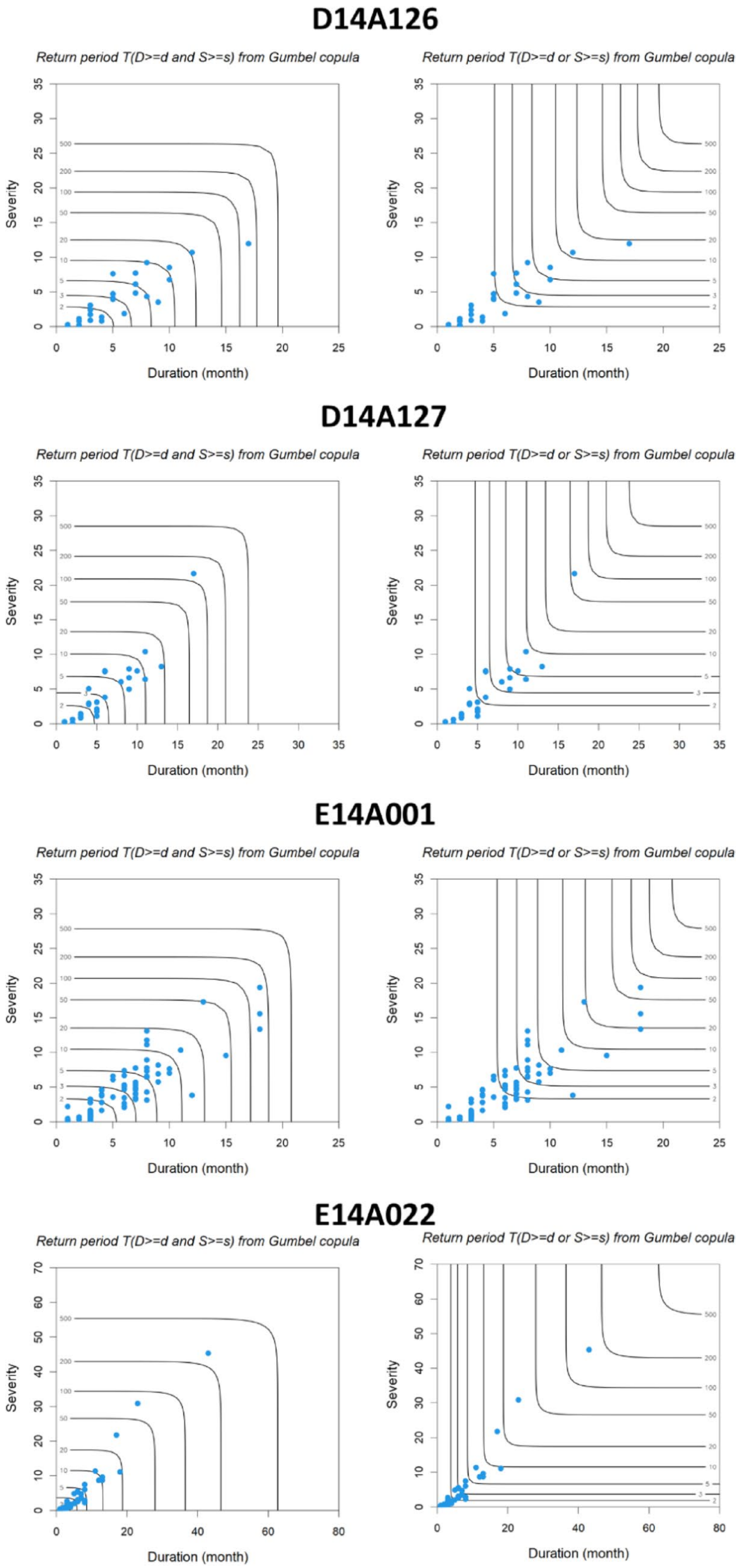
small portions of east side. It is valuable to highlight that distribution of T_{DS} in 10, 20 and 50-year seem very similar. T'_{DS} has (39.21–44.51 years) is recorded as higher in west side, and some parts of east side. In 100-year return period the results have indicated that, univariate drought duration (16.94–49.89 months) is captured with higher distributions of longer durations in west sides. Severity is observed with a distribution range of 15.83–100.11 and particularly higher in west sides and a small portion of east. T_{DS} is recorded with a distribution range of 114.15–138.28 years in this time it has shown a similar distribution pattern to 50-year and most of the east side is recorded with shorter return periods. T'_{DS} with a range of 78.32–88.97 years is detected as higher in west, many parts of north and east sides which is very analogous with previous cases of T'_{DS}. In 200-year return period, drought duration which has become very alike to previous return period′s univariate drought duration is distributed to basin with total range of 18.63–66.89 months and it is noted with longer durations mainly in west sides. Severity with a total range of 18.49–170.88 is recorded as higher in west and small portions of east sides indicating a very similar distribution is to previous return period′s univariate drought severity apart from some small differences. Bivariate return periods of T_{DS} (228.39–276.86 years) and T'_{DS} (156.54–177.89 years) are also demonstrated a similar distribution behavior to 100-year period′s bivariate return periods indicating that the basin is influenced as same even with 100-year difference except at distribution ranges. In 500-year return period, univariate drought duration (20.79–95.29 months) has same distribution 200-year return period but some parts at middle and south becomes darker. Univariate drought severity (22.05–325.49) stayed nearly same except at some small parts in east and middle becoming lighter. T_{DS} (571.08–692.61 years) and T'_{DS} (391.21–444.65 years) is observed as same with 200-year return period except at some small local differences. However, the distribution ranges have become more than twice of 200-year return period in both bivariate return periods of T_{DS} and T'_{DS}.

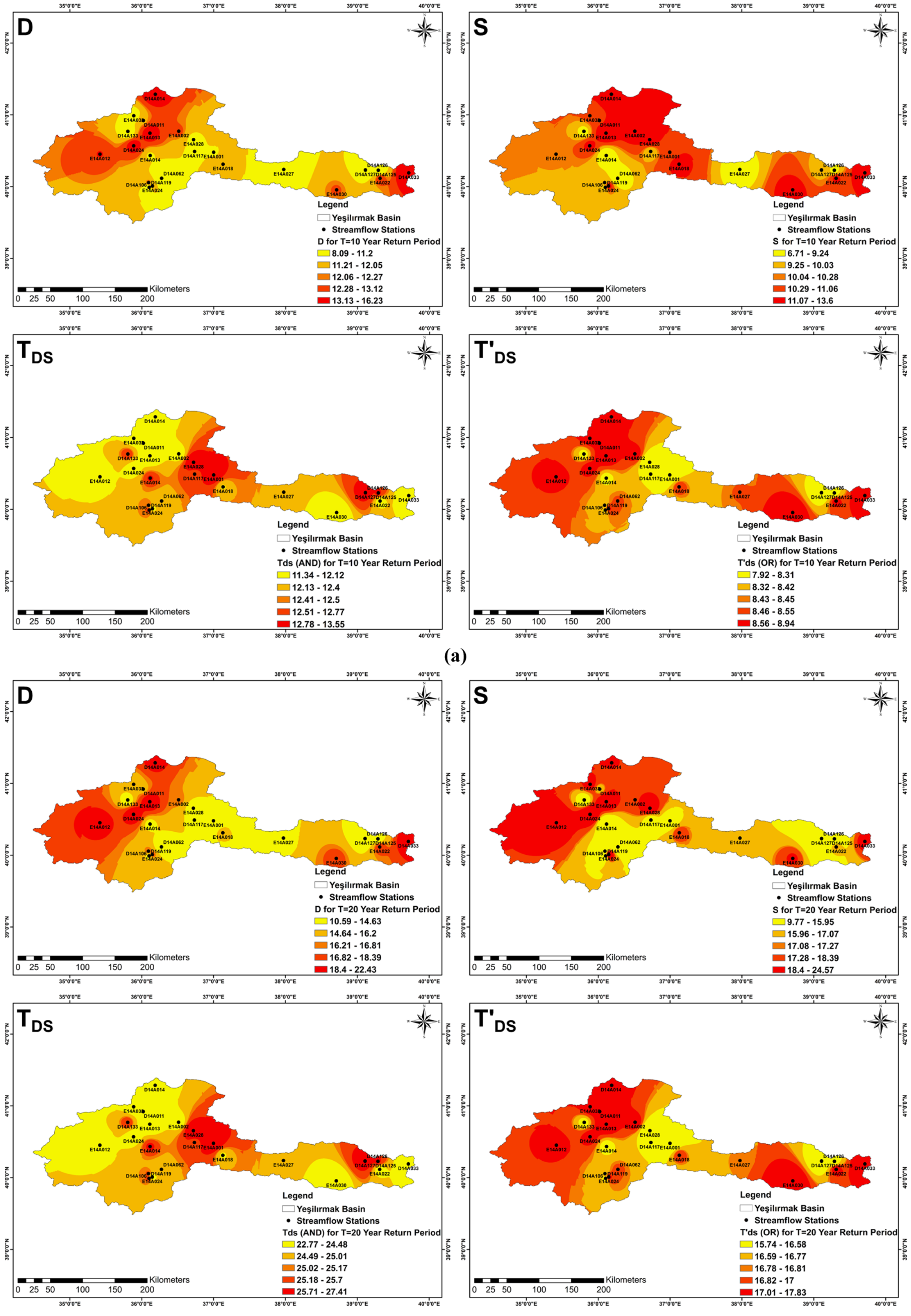
For an effective water resources management, modelling the return periods of drought characteristics as conditional provides effective predicting and assessing current and future drought conditions with a wider perspective. To do so, as it was shown by Eqs. 16 and 17 previously, return period of drought characteristics are modelled as conditional using best copula functions. Here firstly, the conditional return period of drought duration given drought severity exceeding s′ is investigated for all stations. As an example, case for this condition (s′), the conditional return period of the drought duration exceeding 10 months given drought severity exceeding 6 is determined as 29.74 years in D14A011 station, 179.29 years in D14A062 station, 19.06 years in E14A013 station and 31.87 years in E14A022 station. The return periods of this selected case is ranged from 16.45 years to 179.29 years in all the basin. Secondly the conditional return period of drought severity given drought duration exceeding *d*′ is investigated. To give an example case, the conditional return period for the drought severity exceeding 10 given drought duration exceeding 6 months is observed as 24.60 years in D14A011 station, 105.37 years in D14A062 station, 16.19 years in E14A013 station and 27.27 years in E14A022 station. For this case the return periods have the range of 15.25–105.36 years in all the basin. The conditional return periods is illustrated by Fig. 12. It is very obvious that the return periods of *d*′ condition or s′ condition, the stations with lower return periods state more risk of drought. Addition to this point view, since the return period range of selected case in *d*′ condition gave lower return period range than selected case of s′ condition, then it contains more risk of drought.

4.8 Discussion

The objective of this study is to construct multivariate distribution functions for drought characteristics using copulas in Yeşilırmak Basin. Drought events were predicted by SDI for the 3-month time scale. Trends in SDI-3 series of all stations are investigated using Mann–Kendall test, Spearman's Rho test and Wilcoxon test. Drought severity and duration series

Fig. 10 The bivariate return period cases of T_{DS} (AND) (left) and T'_{DS} (OR) (right) for drought characteristics

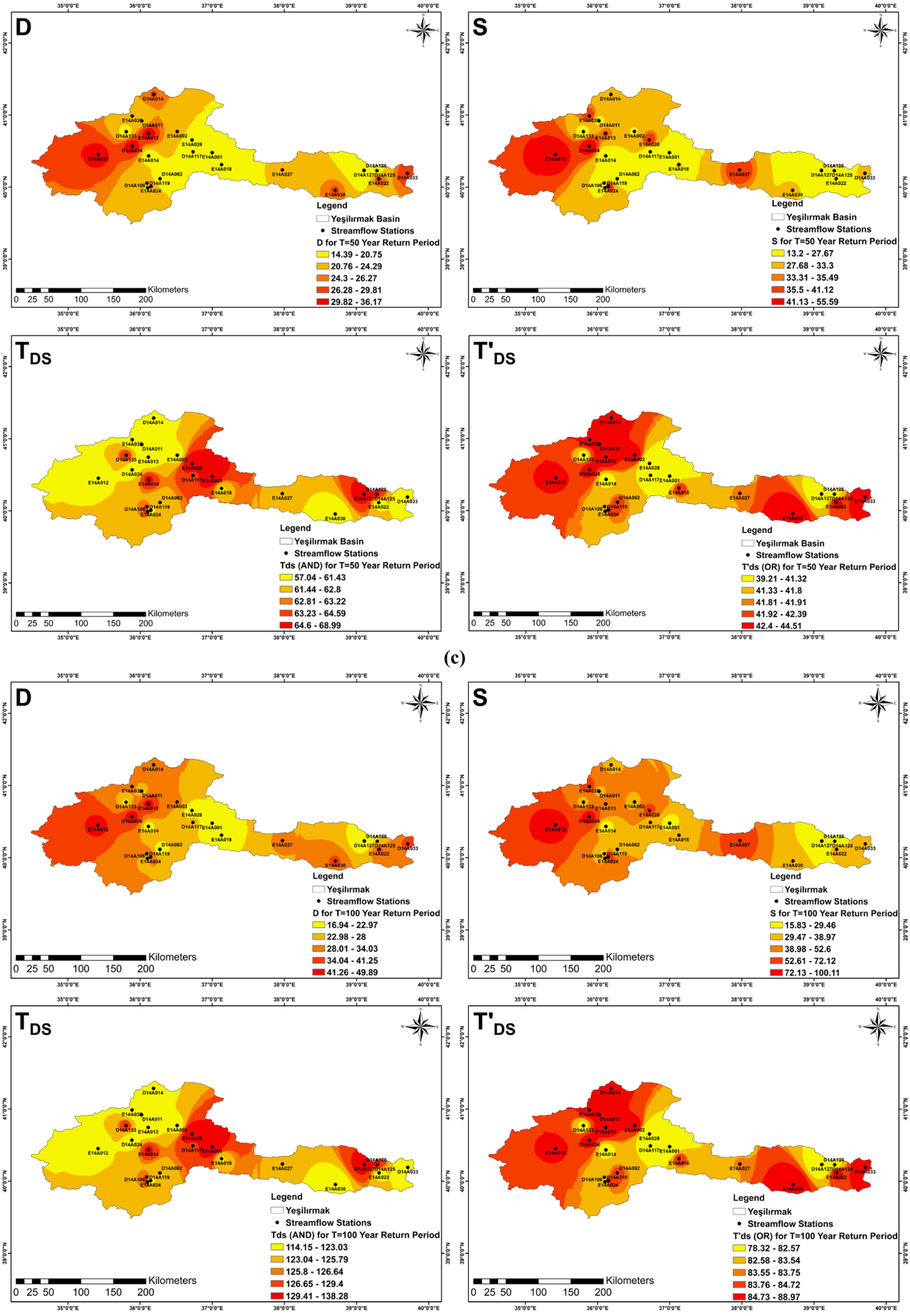




(a)

(b)

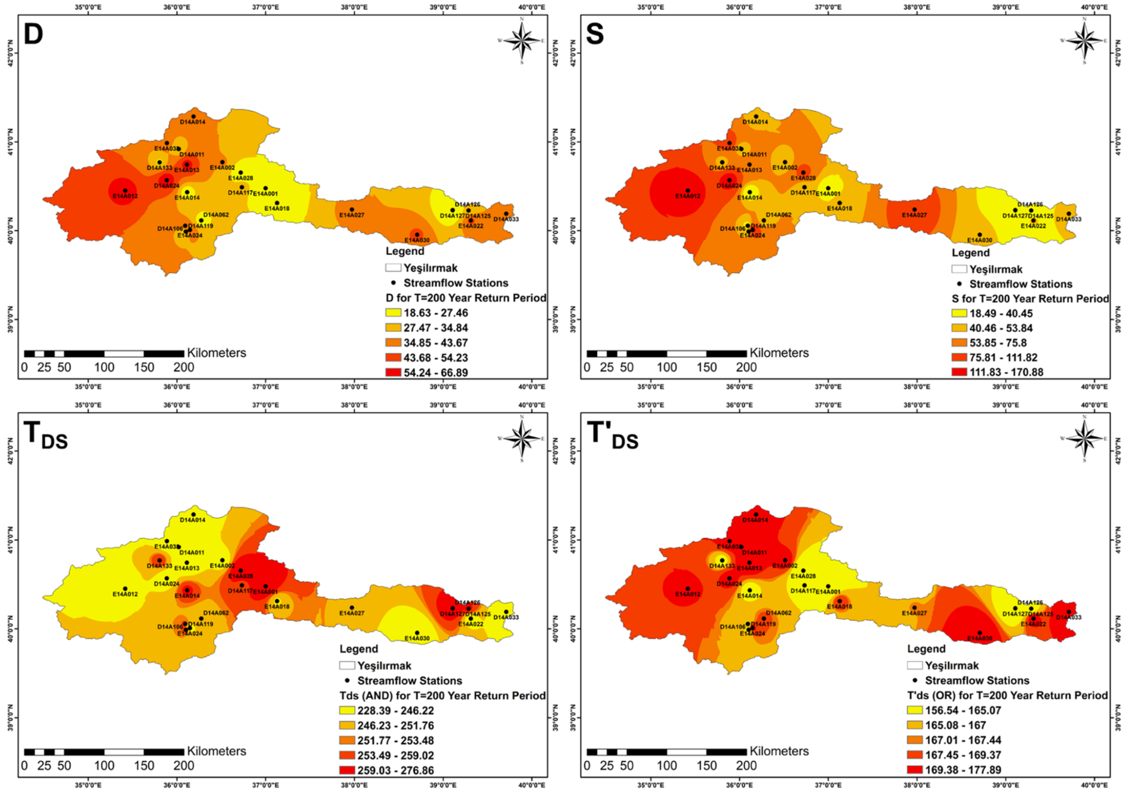
Fig. 11 Spatial distribution of univariate drought duration (D), drought severity (S), and bivariate return periods of T_{DS} (AND), T'_{DS} (OR) based on **a**10-year, **b** 20-year, **c** 50-year, **d** 100-year, **e** 200-year, **f** 500-year in Yeşilırmak Basin



(c)

(d)

Fig. 11 (continued)



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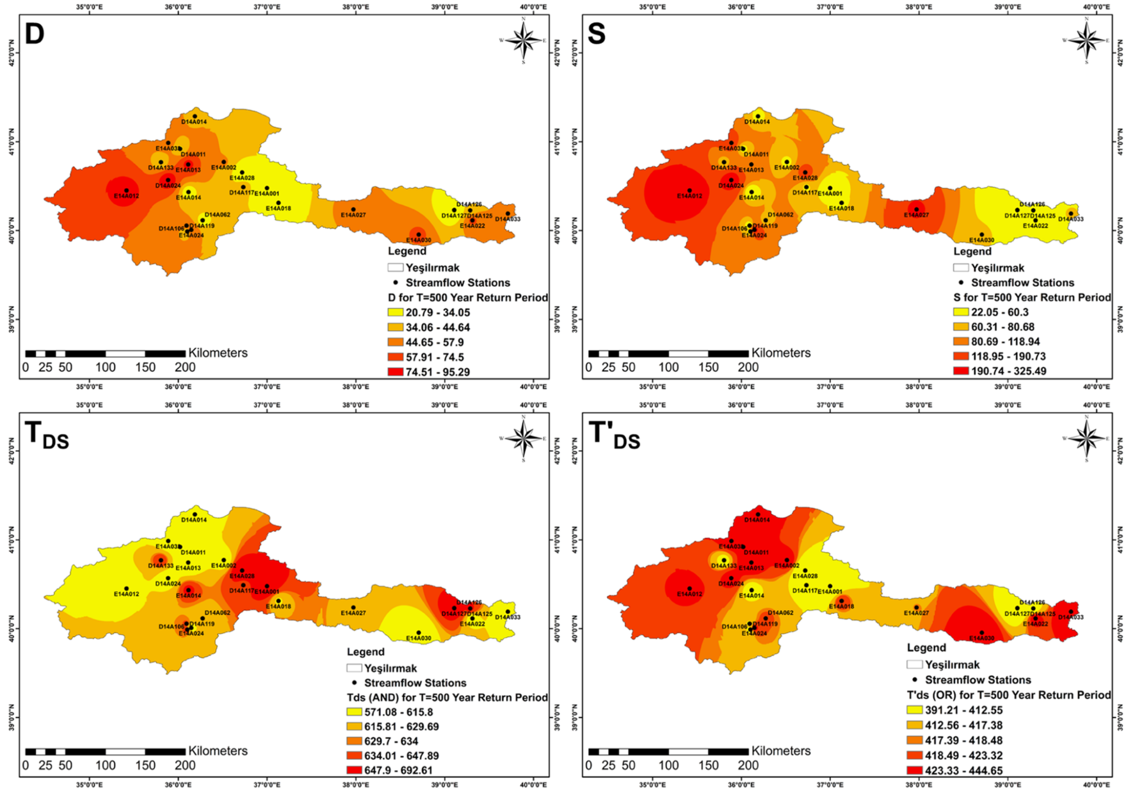
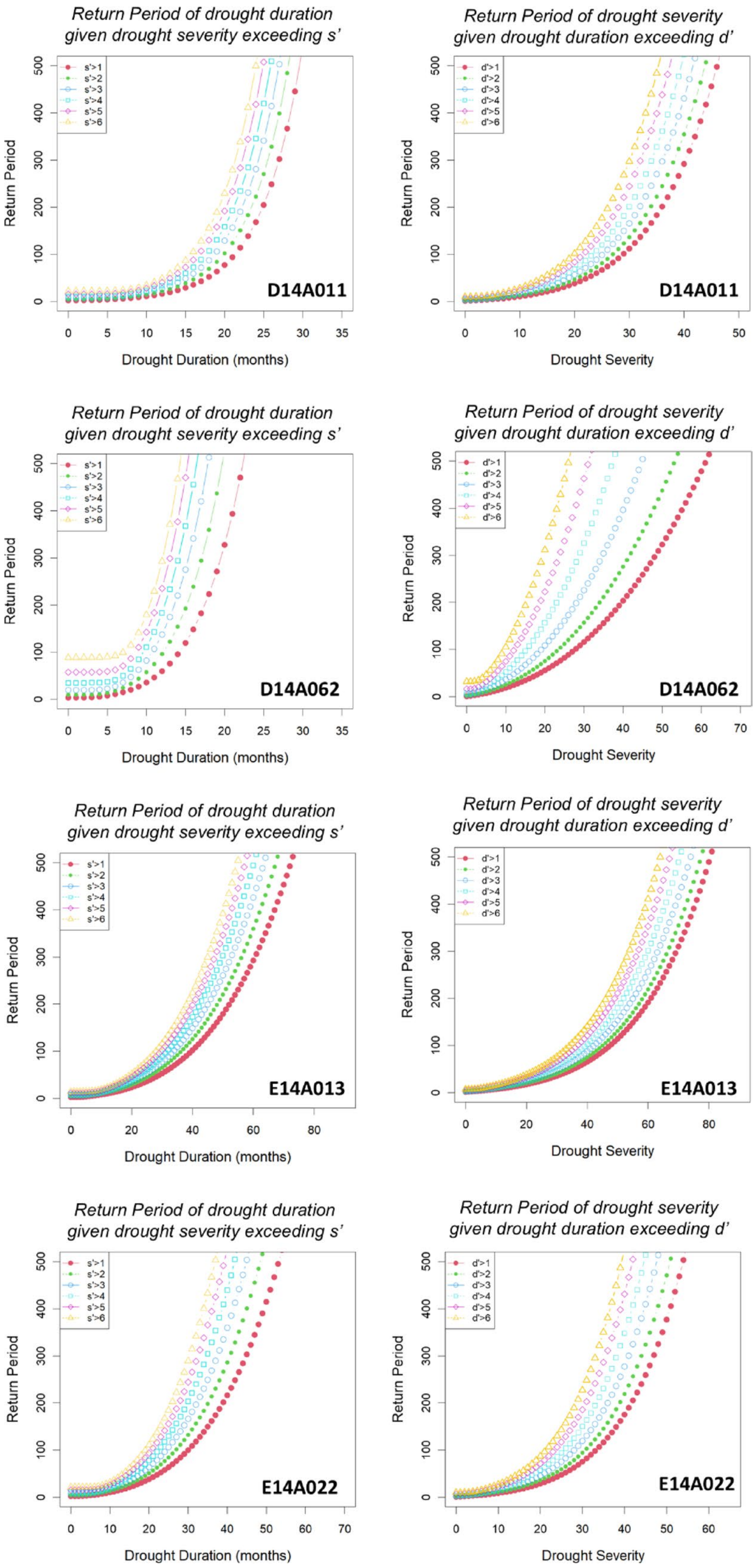


(f)

Fig. 11 (continued)

Fig. 12 The conditional return periods of s′(left) and d′(right) for drought characteristics



are determined by Yevjevich's Run Theory and dependence between them is examined by Kendall's τ. The best marginal distributions of drought severity duration series are found using 6 distribution types and a set of goodness of fit tests. The best copulas are found among 10 types of copulas based on tail dependence and goodness of fit tests. From the best copulas conditional distributions and univariate, bivariate and conditional return periods for drought characteristics are determined.

The results of this study have put forth that there has been a decreasing trend dominance in the basin. In a big number of stations (15 stations) which is more than half of the stations, significant decreasing trends are captured for the level of $\alpha = 0.01$. This is the first signal of potential droughts they may take place in the basin in the future as SDI values are continuously decreased. Recently, in some drought monitoring studies particularly for Yeşilırmak Basin such as (Katipoğlu et al. 2022; Yuce et al. 2023) the presence of decreasing trends are noted. Via Kendall's τ whose range differs in the span of $[-1, +1]$ (Mehdizadeh et al. 2023), a positive strong correlation are found between drought characteristics in all stations. In all the basin the correlations are ranged from 0.719 to 0.907. Earlier, a strong agreement between drought duration and severity are found by researchers (Mirabbasi et al. 2012; Tosunoglu and Can 2016; Vazifehkhah et al. 2019; Esit and Yuce 2023; Shaw and Chithra N R 2023; Deger et al. 2023, 2025). Marginal distribution analysis has shown that drought severity and duration series are modelled well by Lognormal distribution while severity series are modelled by Weibull and Lognormal distributions. In some studies (Tosunoglu and Can 2016; Tosunoğlu and Onof 2017; Vazifehkhah et al. 2019; Esit and Yuce 2023; Deger et al. 2023, 2025; Ahmad et al. 2024), these distributions are found with a good performance in the modelling of drought characteristics. In the current paper, it is seen that multivariate distribution functions are mostly (23 stations out of 24 stations) constructed via Gumbel copula which is found the best copula many times considering firstly tail dependence and secondly a set of goodness of fit tests. Gumbel copula has been noted with a good performance in many studies (Dodangeh et al. 2017; Nabaei et al. 2019; Hasan and Abdullah 2022; Deger et al. 2023, 2025; Suo et al. 2024; Palagiri et al. 2024; Meimandi et al. 2024). Therefore, the suggested findings of the current paper seem compatible. Besides, the current paper presents a detailed risk assessment of hydrological drought characteristics considering spatial and temporal variabilities especially in different return periods.

5 Conclusion

In this study, multivariate distribution functions of hydrological drought characteristics are modelled using copula functions for predicting and assessing conditional probabilities, univariate, bivariate and conditional return periods. Overall, the following conclusions are made based on our findings:

- Analysis of SDI-3 values across all stations revealed the occurrence of numerous critical drought events, both historically and recently, within the basin. The presence of these events is associated with a decreasing trend in SDI-3 prevalence. Decreasing trends, both statistically significant and non-significant, were frequently observed in the SDI-3 time series of the stations.
- Yevjevich's Run Theory has allowed to predict drought severity and duration series properly as the dependence between them that is checked by Kendall's τ has been detected with strong positive correlations.
- Marginal distribution analysis has put forth that Lognormal and Weibull distributions are good at modelling drought characteristics in Yeşilırmak Basin. Normal and logistic distributions are noted with a bad performance.
- Gumbel copula is found as the best copula for the modelling of hydrological drought characteristics based on tail dependence and goodness of fit tests. The upper tail dependence has provided a safer choice as it has a critical role especially in care of preventing uncertainty.
- Conditional probability approximations of hydrological drought characteristics by the best copulas are provided to assess drought events under different conditions. The findings of the study are suggested that a basin or a region can be affected from droughts with different conditions of s' and d' (different magnitudes in severity and different length in duration). For making a sensitive drought monitoring work, it is better to know the characteristics of the study area. In the current paper random events are selected to show determination and evaluation of the events.
- The univariate and bivariate return periods of hydrological drought characteristics have shown that the bivariate return periods (T_{DS} and T'_{DS}) are given a safer information than univariate ones as drought is multivariate extreme disaster. To see the potentials of univariate and bivariate return periods over the basin, numerical findings must be associated with spatial extensions under different conditions. No doubt that the longer return periods having a smaller risk compared to shorter return periods. In addition to this the ranges of any evaluation criteria (D, S, T_{DS} and T'_{DS}) must be followed for consistent predictions. Based on the findings, the basin has

risk of drought with changeable regions under different conditions. Similar to conditional probability cases, conditional return periods also provided a risk assessment for the basin. Compared to univariate and bivariate return periods, in conditional return periods there is a flexibility as severity or duration conditions can be set up to desire considering exceeding conditions.

- From the findings, the study has put forth that drought events must be considered as multivariate events and can be modelled by copulas which are powerful tools. Therefore, the basin which is very critical for the country's development, economy, sustainability, water resources management, historical and cultural heritage must have a stronger and effective drought action plan. The study is expected to be helpful for researchers, local authorities, government agencies for monitoring the drought events and keeping the water and environment stable. Therefore, the integration of SDI-3 with copula functions provides a significant development for proactive drought monitoring and response policies, particularly in Turkey where seasonal water stress and climate variability may influence water resources.
- Although many significant insights have been acquired by this paper, the research is limited with analyzing hydrological drought characteristics by suggested methodology and stationary assumption. Due to climate change, sensitivity of water related structures and effective drought mitigation, future studies could consider non-stationary approaches with compatible methods such as indices, considering droughts as multivariate events.

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Code availability Not applicable.

Declarations

Conflict of interest The authors declare no competing interests.

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Informed consent This study did not include any human participants or animals.

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