Resilient Graph Partitioning for Quantum Networks

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***Abstract*—Quantum entanglement is important for quantum communications for long range transmission. Such long transmission is achieved through entanglement swapping, a method monopolizing considerable resources as the quantum network topology expands. Therefore, in this work we propose a method to reduce the resources required for entanglement swapping using network partitioning while ensuring resiliency. This method involves the fidelity of distributing entangled states that depends on the source, the link and the bell state analyzer utilized to quantify the required fidelity for a functioning network. By partitioning the network to redundant sub-graphs, we show the advantage of such approach in reducing the required functional fidelity.**

***Index Terms*—Network Partitioning, Resiliency, Quantum Network.**

I. INTRODUCTION

Quantum information science is an important field of research and technological development, driving advances in communication networks. Its impact is noticeable in key areas such as cryptography [1] offering security enhancement, sensing [2] for a precise measurement, and computing [3] with its promise to outperform classical computation for certain tasks. This rapid maturation of quantum technologies have attracted the attention of the telecommunication community, especially within the context of emerging 6G networks[4, 5, 6, 7], attention further stimulated by the quantum internet concept with its premices on quantum routine protocol [8].

Such quantum internet require transferring efficiently the quantum information across a network, a task that is much challenging than its classical counterpart due to the unique properties of quantum data, such as superposition and entanglement. Similar to circuit switching in traditional networks, one might envision a quantum switcher dedicated to store and route the quantum information to its end destination. Unfortunately, quantum memories are still at their early stage facing limited coherence times. As a result, memoryless quantum networks are considered as the best fit at the time being, where the quantum information is transmitted using protocols such as quantum teleportation or entanglement swapping transmission require entangled state to insure immediate transmission, which requires distributing entangled states between the end users.

The Bell state distribution chain is one method to distribute entangled states through a quantum network. However, as the produced entangled states endure losses during their travelling, switch nodes are needed to maintain a long transmission line between two end nodes. Given that the option of storing quantum information was sidelined, the network must operate continuously at high regime to ensure a high quality of quantum data transfer, which is energivorous. To adress this issue, one solution is to allocate a chain of nodes from the network to transfer the quantum information for a specific demand, while ensuring the redundancy by reserving additional independent paths, as redundancy is key component of network resiliency.

Thus, in this study, we introduce resilient graph partitioning for a memoryless quantum network to optimize resource allocations while maintaining a functional network. First, in the methodology section, we conceptualize the network as multi-node system helping to evaluate the fidelity, which is a critical measure reflects of quantum information being transmitted. This approach asses how the networks performs under different conditions.

In the next section, we introduce the definition of a redundant sub-graphs and the method for partitioning triangular lattice topologies into redundant complementary sub-graphs. We then demonstrate that this approach with the set of sub-graphs uses fewer resources to reach the required fidelity for an optimal functioning compared to the original graph. Finally, in the last section, we evaluate the performance of the partitioning algorithm as the number of nodes constituting the quantum network increases by applying the partitioning algorithm on randomized generated connected graphs.

II. RELATED WORKS

In previous studies, network partitioning has been used to address the multicontrol placement problem, decreasing congestion and computational effort, minimizing cost function and lowering the communication latency, among other benefits. Researchers have investigated multi-controller placement problem in SDN and revealed a novel approach with network partition technique [9]. With this new technique, they had a chance to decrease the complexity of the controller placement and apply the performance objectives to the sub-topologies instead of the whole topology.

Literature [10] reveals a technique to solve the dual balanced graph partition problem, minimizing the cut size while ensuring both vertex and edge balance. In another study [11], researchers have developed a clustering algorithm which distributes the network nodes into clusters. Each cluster has a head node, and the remaining nodes in the cluster send the information to the head node. The head nodes handle data integration and forwarding, leading to lowering the energy consumption of the network. Literature [12] introduces a new adaptive network partition for multi intersection traffic signal control. It proposes two network partition strategies based on graphical neural network (GNN) and Monte Carlo tree search (MCTS), with MCTS demonstrating better performance compared to GNN in large scale networks. Network partition problem using k-means algorithm was addressed in [13], showing a decrease in the computational effort of auxiliary service analysis in DNs. Meanwhile, In [14], multilevel algorithms were used to distribute unstructured meshes onto parallel computers, minimizing a cost function based on a model of communication networks. The literature [15], applied network partitioning to reduce communication latency between controllers and switches.

In the context of quantum networking, topology design has great importance for entire network as the large scale quantum network deployment increases, where literature [16] investigates different topology architectures that are suitable for the future quantum internet. Although there are studies on network partition in the literature, to our best knowledge, no study has minimized the required fidelity value by applying network partitioning to quantum networks.

III. METHODOLOGY

As for any network conceptualization, we model the quantum network as a link connected multi-node system where each component ensures a particular functionality. In this work, we restrain our study to the most important elements to design the Bell state distribution chain:

1) Quantum node: also known as the quantum source node where the entangled states are generated.

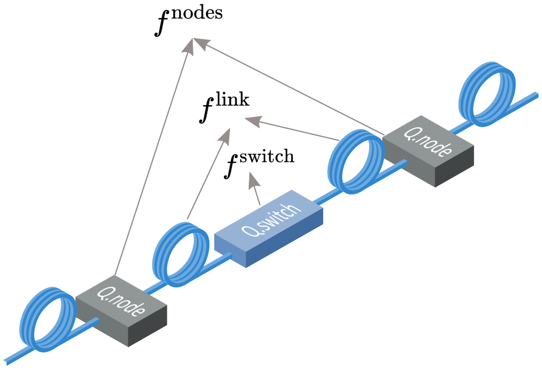


Fig. 1: Unit description of the basic components of a quantum network

2) Quantum switch: in this component, the entangled states are processed using a Bell state analyzer.
3) Quantum link: connecting component between the quantum node and the quantum switch.

Each of these components contributes to reduce the quality of the quantum state associated respectively to the swapping fidelity f^{nodes}, the link fidelity f^{link} and the switch fidelity f^{switch} as depicted in fig. 1. Very often, distributing quantum states through the chain of Bell state analyzers comes with a probability encompassing the aforementioned fidelities stated as [17, 18]

$$P_{\text{dist}} = \prod_{i=0}^{n_h} f_i^{\text{node}}.f_i^{\text{link}}.f_i^{\text{switch}} = \prod_{i=0}^{n_h} f_i \tag{1}$$

with n_h the number of hops, also associated to the number of source nodes as $n_d = n_h + 1$. So for any n_h, one might build a connected graph with maximum number of node of n_d. For simplicity, in the following sections, we consider all the units in the Bell state analyzer chain presenting the same physical characteristics, leading to $\forall i, j \in n_d$ and $i \neq j$, $f_i = f_j$. Under this condition, n_h is expressed as following

$$n_h = \left\lfloor \frac{log(P_{\text{dist}})}{log(f_i)} \right\rfloor \tag{2}$$

A. Quantum Network Accessible Connected Sub-graphs

For each graph, it is essential to keep in consideration its operationality. Since P_{dist} is a user defined probability of entangled states that must be achieved to fulfill a designated task, here we check at which required fidelity $f_{i,req}$ one has access to the total graph network.

To do so, without loss of generality, we set an example with the triangular graph lattice T_3 and we fix $P_{\text{dist}} = 0.900$. By varying f_i, we establish the accessible connected sub-graphs generated from the original triangular graph, starting with sub-graphs of $n_d = 2$ for $f_i < 0.948$ and with total accessibility of the triangular graph of $n_d = 10$ at the required operational fidelity $f_{i,req} = 0.989$. We illustrate these results in fig. 2 where we count all possible accessible connected sub-graphs with respect to f_i.

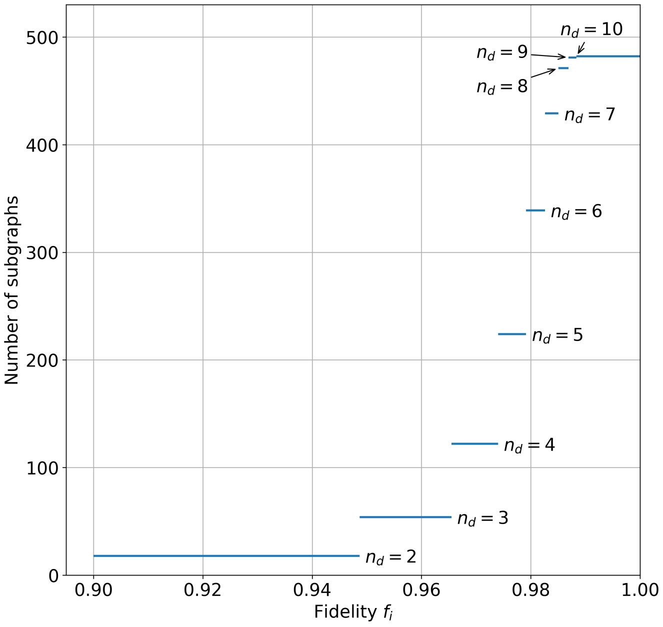


Fig. 2: Total number of sub-graphs as function of fidelity f_i

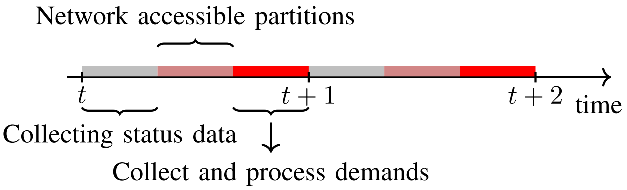


Fig. 3: Time scheduling for demands processing with a time frame divided in three consecutive parts: First collecting the status data of the devices constituting the network, then filtering the accessible partitions with respect to the status data, and finally the collected demands are processed.

IV. NETWORK PARTITIONING PERSPECTIVE

In this section, we highlight the importance of network partitioning in reducing the resources allocated for distributing entangled states.

For a memory-based quantum network, storing entangled states in switches, and release them later on would be an ideal for distributing entangled states [19, 20]. But in the current state of the art, the quantum memories have not reached yet the deployment stage at the large scale. Instead, we suggest a time schedule plan employing network partitioning for a memoryless quantum network as depicted in fig. 3. This time schedule plan proceeds as following:

1) Collecting data about the network status to estimate the swapping, link and switch fidelities composing the physical network.
2) Filtering and selecting all sets of accessible sub-graphs generated from partitioning the physical network.
3) Collecting and processing the demands with respect to the available sets of sub-graphs. At this stage, parallel processing might be possible if the two or more demands fit in a non-overlapped sub-graphs.

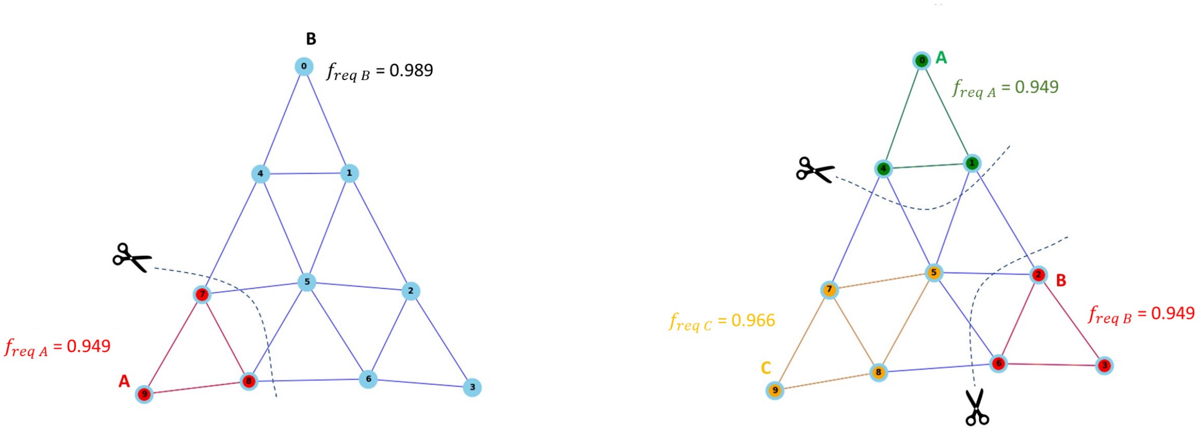
To be able to process demands, we need a preestablished collections of sub-graphs on which we assign the corresponding demand treatment. These sub-graphs must show redundancy, meaning that each node in the sub-graph can reach another node with at least two disjoint pathways. Thus, we focus our efforts in providing details about the mathematical description and the algorithm to generate redundant sub-graphs.

A. *Redundant sets of sub-graphs*

Let $\{g_i\}$ be a set of of sub-graphs after partitioning a graph G. If the graph G accepts j distinguished partitioning scenarios, then this partitioning forms the set of sets $\{\{g_i\}_j\}$. For any set of end-to-end vertices $\{V_a,\ V_b\} \in g_i$, $\exists$ at least two disjoint paths P_1 and P_2 to ensure redundancy: $P_1 \cap P_2 = \{\{V_a,\ V_b\}_V;\ \emptyset_E\}$. From previous work, it has been already proven that a connected graph g_i with at least 3 vertices is 2-connected if and only if, for every pair of vertices V_a and V_b in g_i, there exist two internally disjoint paths P_1 and P_2 between V_a and V_b". As a result, to certify that a sub-graph is redundant, we remove one node and verify that the sub-graph is still connected.

The partitioning algorithm is depicted in Algorithm 1. It takes as an input an adjacency matrix of a graph (in our case a triangular lattice T_3) and output a list of possible scenarios i.e a list of set of sets of complementary sub-graphs. It starts by collecting the nodes in the graph G, then creates arbitrary sub-graphs using the function `.subgraph` from Networkz library. From this collection of sub-graphs, we select the 2-connected sub-graphs, then we create the complementary sub-graph which go through the same process and register the sub-graph with its complementary as a scenario. Some of the complementary sub-graphs might be identified as a connected sub-graph with at least 6 nodes, it occurs that partitioning this complementary sub-graphs might also generate 2-connected graphs, resulting in other possible scenarios.

In fig. 4a and fig. 4b are presented the two possible partitioning scenarios for a triangular lattice graph T_3 of $n_d = 10$. The first partitioning unveils two redundant sub-graphs of which the required fidelities for functional sub-graphs are reduced to $f_{req\ A} = 0.949$ and $f_{req\ B} = 0.983$. Meanwhile three redundant sub-graphs are accessible for fidelities of $f_{req\ A} = 0.949 = f_{req\ B}$ and $f_{req\ C} = 0.966$. All of these redundant sub-graphs scenarios mobilize less resources compared to the initial functional fidelity of the graph G, $f_{req\ G} = 0.989$. Additionaly, we summarize these results in fig. 5 with the blue dotted marks, meanwhile we display also the outcomes for the T_4 triangular lattice given in red diamonds marks. The dashed lines highlight the difference in the required fidelity for a functional network between the first case partitioning with 2 sub-graphs and the last ones with 3 for T_3 and 4 for T_4. We complete this analysis by providing the number of possible scenarios for each number of partitioned graph described in Table I.



(a) (b)

Fig. 4: Two possible scenarios for T_3 tirangular lattice graph. figure (a) represents the partitioning resulting a two redundant sub-graphs of functional fidelities of $f_{req\ A} = 0.949$ and $f_{req\ B} = 0.983$. Meanwhile, in figure (b) the network is partitioned in three sub-graphs of functional fidelities $f_{req\ A} = 0.949$, $f_{req\ B} = 0.949$ and $f_{req\ C} = 0.966$.

Algorithm 1 Algorithm for Finding redundant sets of sub-graphs

```
Input: A graph $G$
Output: Set of possible scenarios
function PARTITIONING($G$)
Initialize $nodes \leftarrow$ list of nodes in $G$
  for $r$ from 3 to $length(nodes)$ do
    for $subset$ in all combinations of nodes of size $r$ do
      $subgraph \leftarrow G.subgraph(subset))$
      if $subgraph$ is connected and node connectivity of $subgraph \geq 2$ then
        $complement\_subgraph \leftarrow complementary(G, subgraph)$;
        if number of nodes in $complement\_subgraph > 0$ and $complement\_subgraph$ is connected and node connectivity of $subgraph \geq 2$ then
          $scenarios \leftarrow subgraph$
          $scenarios \leftarrow complementary\_subgraph$
          if number of nodes in $complement\_subgraph > 5$ and $complement\_subgraph$ is connected then

            function PARTITIONING($complement\_subgraph$) {Call again function Partitioning recursively}
return $scenarios$
```

Number of partitions in a graph	1	2	3	4
Number of scenarios in T_3	1	3	3	/
Number of scenarios in T_4	1	16	23	4

TABLE I: number of scenarios in each possible partition for triangular lattice graphs T_3 and T_4.

V. PERFORMANCE TEST FOR RANDOM GRAPHS

Finally, we conduct an investigation on the performance of our partitioning algorithm over random connected graphs. To do so, we use the Erdős–Rényi model from the Networks library to generate randomized connected graphs of n_d number of nodes. For each selected number of nodes, we run the partitioning algorithm over a 1000 random graphs, and evaluate the average time representing the average performance (cf fig. 6). This sample size gives us a reliable assessment of the algorithm's behavior and enables us to lessen the influence of any deviations.

To better understand the nature of this scaling, we apply a linear fitting to the logarithm of the average runtime as a function of n_d. Through this fitting, we are able quantify the growth rate of the algorithm's computational complexity, revealing that the performance exhibits an exponential relationship with the size of the graph. More precisely, a trend that can be represented as $\mathcal{O}(e^{kn_d})$ where $k = 1.69$ is a constant obtained from the linear fit.

VI. CONCLUSION

Insuring a high fidelity is cost-efficient for the network. So in this work, we have proposed a method based on network partitioning to reduce the resources mobilized for the network by lowering the required fidelity. In real world application, one

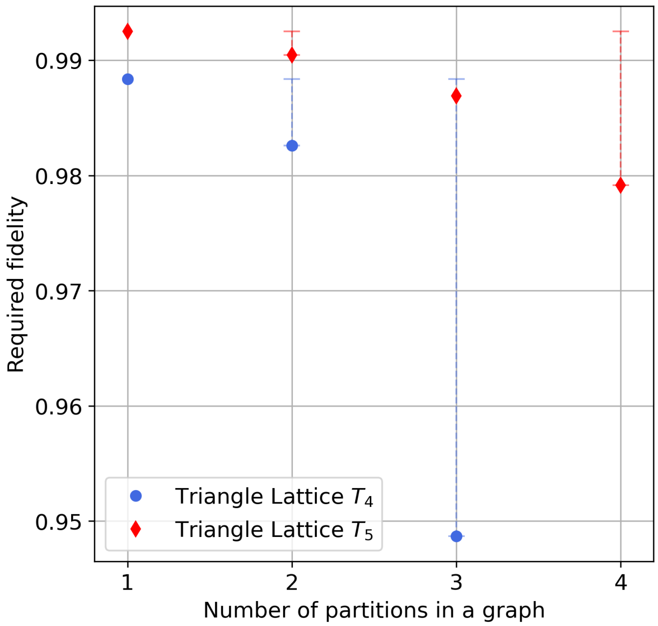


Fig. 5: Fidelity as function of the number of sub-graphs in a case.

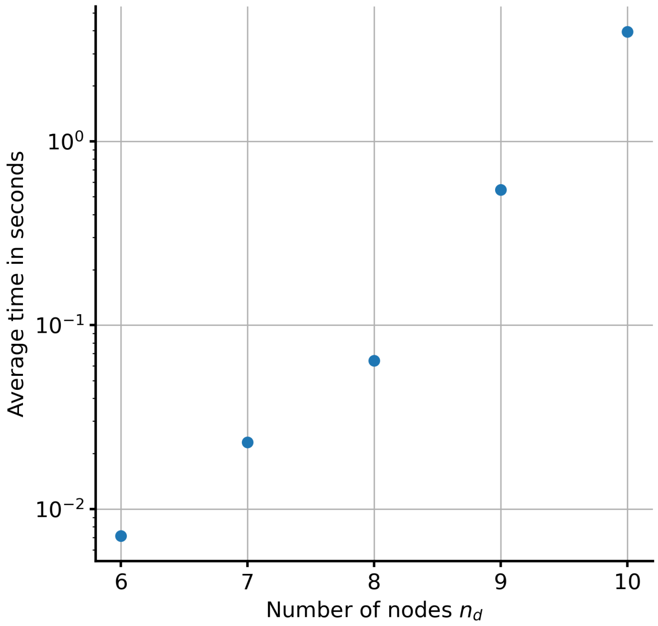


Fig. 6: Average time in seconds as function of the number of nodes n_d.

needs to aim at reaching the required fidelities for the desired scenario instead.

Although this study demonstrates its effectiveness on triangular lattice graph, it does not prevent the user of the algorithm to test other topologies, as the conditions imposed are topology-independent.

Future work would include improvement of the partitioning algorithm to enhance its performance on large networks for more scalability. Additionally, we plan to investigate the parallel processing of the demands, thus boosting time efficiency and making the algorithm more practical for real-time applications.

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REFERENCES

[1] A. Rubenok, J. A. Slater, P. Chan, I. Lucio-Martinez, and W. Tittel. Real-world two-photon interference and proof-of-principle quantum key distribution immune to detector attacks. *Physical Review Letters*, 111(13), September 2013.

[2] B. J. Lawrie, P. D. Lett, A. M. Marino, and R. C. Pooser. Quantum sensing with squeezed light. *ACS Photonics*, 6(6):1307–1318, 2019.

[3] P. Drmota, D. P. Nadlinger, D. Main, B. C. Nichol, E. M. Ainley, D. Leichtle, A. Mantri, E. Kashefi, R. Srinivas, G. Araneda, C. J. Ballance, and D. M. Lucas. Verifiable blind quantum computing with trapped ions and single photons, 2023.

[4] Chonggang Wang and Akbar Rahman. Quantum-enabled 6g wireless networks: Opportunities and challenges. *IEEE Wireless Communications*, 29(1):58–69, 2022.

[5] Anutusha Dogra, Rakesh Kumar Jha, and Shubha Jain. A survey on beyond 5g network with the advent of 6g: Architecture and emerging technologies. *IEEE Access*, 9:67512–67547, 2021.

[6] Riccardo Bassoli, Holger Boche, Christian Deppe, Roberto Ferrara, Frank HP Fitzek, Gisbert Janssen, and Sajad Saeedinaeeni. *Quantum communication networks*, volume 23. Springer, 2021.

[7] Patrick Seeling, Frank Fitzek, and Fabrizio Granelli. *Computing in Communication Networks*. 06 2020.

[8] Stephanie Wehner, David Elkouss, and Ronald Hanson. Quantum internet: A vision for the road ahead. *Science*, 362(6412):eaam9288, 2018.

[9] Guodong Wang, Yanxiao Zhao, Jun Huang, Qiang Duan, and Jun Li. A k-means-based network partition algorithm for controller placement in software defined network. In *2016 IEEE International Conference on Communications (ICC)*, pages 1–6. IEEE, 2016.

[10] Chengjun Liu, Zhuo Peng, Weiguo Zheng, and Lei Zou. Fsm: A fine-grained splitting and merging framework for

dual-balanced graph partition. *Proceedings of the VLDB Endowment*, 17(9):2378–2391, 2024.

[11] De-gan Zhang, Si Liu, Ting Zhang, and Zhao Liang. Novel unequal clustering routing protocol considering energy balancing based on network partition & distance for mobile education. *Journal of Network and Computer Applications*, 88:1–9, 2017.

[12] Jinming Ma and Feng Wu. Learning to coordinate traffic signals with adaptive network partition. *IEEE Transactions on Intelligent Transportation Systems*, 2023.

[13] M Biserica, G Foggia, E Chanzy, and JC Passelergue. Network partition for coordinated control in active distribution networks. In *2013 IEEE Grenoble Conference*, pages 1–5. IEEE, 2013.

[14] Chris Walshaw and Mark Cross. Multilevel mesh partitioning for heterogeneous communication networks. *Future generation computer systems*, 17(5):601–623, 2001.

[15] Eiichiro Kawaguchi and Norihiko Shinomiya. Network partitioning algorithm for minimizing communication latency in sdn. In *TENCON 2018-2018 IEEE Region 10 Conference*, pages 1529–1532. IEEE, 2018.

[16] Ruozhou Yu, Rudra Dutta, and Jianqing Liu. On topology design for the quantum internet. *IEEE Network*, 36(5):64–70, 2022.

[17] Yiming Zeng, Jiarui Zhang, Ji Liu, Zhenhua Liu, and Yuanyuan Yang. Entanglement routing design over quantum networks. *IEEE/ACM Transactions on Networking*, 32(1):352–367, 2024.

[18] Yiming Zeng, Jiarui Zhang, Ji Liu, Zhenhua Liu, and Yuanyuan Yang. Multi-entanglement routing design over quantum networks. In *IEEE INFOCOM 2022-IEEE Conference on Computer Communications*, pages 510–519. IEEE, 2022.

[19] Kaushik Chakraborty, Filip Rozpedek, Axel Dahlberg, and Stephanie Wehner. Distributed routing in a quantum internet. *arXiv preprint arXiv:1907.11630*, 2019.

[20] Mihir Pant, Hari Krovi, Don Towsley, Leandros Tassiulas, Liang Jiang, Prithwish Basu, Dirk Englund, and Saikat Guha. Routing entanglement in the quantum internet. *npj Quantum Information*, 5(1):25, 2019.